Whatever It Takes? Market Maker of Last Resort and its Fragility

Dong Beom Choi and Tanju Yorulmazer*

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Abstract

We provide a theoretical framework to analyze the market maker of last resort (MMLR) role of central banks. Central bank announcement to purchase assets in case of distress promotes private agents' willingness to make markets, which immediately restores liquidity to prevent disorderly sales. This, in turn, decreases the future need for the central bank to intervene. Here, the central bank can reduce the expected usage of the facility by announcing a large capacity, that is, it can end up buying less ex-post by committing to do more ex-ante. However, this beneficial feature comes with potential downsides. First, the central bank may not achieve the intended outcome due to the possibility of multiple self-fulfilling equilibria, which may arise if it does not intervene with sufficient forcefulness or if market participants have doubts about its commitment. Second, public liquidity provision may crowd out private liquidity if the MMLR access becomes permanent and make the intervention ineffective.

Keywords: market maker of last resort, liquidity, asset purchase program, multiple equilibria, time inconsistency

*Choi: Seoul National University, dong.choi@snu.ac.kr; Yorulmazer: Koç University, tyorulmazer@ku.edu.tr. For helpful comments, we thank Viral Acharya, Franklin Allen, Jess Benhabib, Eduardo Davila (discussant), Douglas Gale, Ilwoo Hwang, Natalya Martynova (discussant), Rafael Repullo, Javier Suarez, Xuan Wang (discussant) and the participants at Bundesbank, CEMFI, Korea University, Seoul National University, the Second Workshop on Financial Intermediation and Corporate Debt Markets by Bundesbank and Humboldt University, 2023 EFA, and the Festschrift in Honor of Douglas Gale. Choi appreciates support from SNU Institute for Research in Finance and Economics. Any errors are our own.

"A properly constructed MMLR must have a large capacity, but might need to do little. ... The classic is Mario Draghi's "whatever it takes," where the ECB provided a backstop for euro area sovereigns but ended up buying nothing" (Cecchetti and Tucker 2021)

"The ECB's efforts rely to an important extent on a bluff, in the sense that nobody knows what would happen if OMT were actually required" (Krugman 2014)

1 Introduction

Central banks traditionally acted as lender of last resort (LoLR) to support financial stability by providing emergency loans to illiquid banks against safe collateral. While this liquidity backstop helped maintain funding stability in the banking sector, they recently had to reinvent themselves as the financial system transitioned from bank-based to market-based, where stable market liquidity became crucial for systemic soundness. In particular, they started acting as a market maker of last resort (MMLR) to address liquidity shortages in specific markets by outrightly purchasing illiquid assets.

Several such interventions achieved remarkable success, where the announcement of MMLR operations immediately stabilized financial markets, even without accompanying actual purchases. A good example would be the Outright Monetary Transactions (OMT) program of the ECB that provided a backstop for Euro area sovereign debt amid the European debt crisis, where Mario Draghi famously promised to do "whatever it takes" but ended up buying nothing. Other examples include the Bank of England's 2009 MMLR operations in sterling corporate bonds and the Federal Reserve's Secondary Market Corporate Credit Facilities (SMCCF) and Municipal Liquidity Facility (MLF) in response to the Covid-19 pandemic. These public backstops instantly reinstated private liquidity upon the introduction and, consequently, the central banks did very little, if anything.[1]

However, skepticism exists regarding the consistent efficacy of these interventions, raising ques-

[1]While authorization was for $750 billion for the SMCCF, the Federal Reserve's holdings of corporate bonds and exchange traded funds only peaked at $14 billion. A number of recent studies document that the Fed's launch of these programs restored market liquidity quickly, even without any actual intervention (Brunnermeier and Krishnamurthy, 2020; Boyarchenko et al., 2021; Haddad et al., 2021; Kargar et al., 2021; O'Hara and Zhou, 2021; Vissing-Jorgensen, 2021).

tions about their robustness (see, e.g., the opening quote by Krugman 2014).[2] In fact, despite being implemented by the same central banks, more recent attempts have proved much less effective. The Bank of England introduced its emergency gilt-buying program in September 2022 to alleviate disruptions in the gilt market, precipitated by the government tax cut announcement and ensuing sell-offs by pension funds. Similarly, the ECB unveiled its Transmission Protection Instrument (TPI) in July 2022 in response to fragmentation in the European sovereign debt market stemming from a shift toward tighter monetary policy. In contrast to previous instances, the announcements of these measures did not have a major impact on market liquidity. Instead, they elicited skepticism regarding the credibility of central banks, as the asset purchases would inevitably lead to a notable expansion of the money supply, contradicting their claims of constraining it to dampen inflation.

How should we reconcile these conflicting outcomes of the prior interventions? What would be the features of a successful facility that achieves the stability objective effectively? Also, should this unconventional asset-purchase operation remain in the central bank's permanent toolkit?[3] To answer these questions, we need a theoretical framework that characterizes the mechanism of MMLR and, more importantly, its possible fragility and downsides. However, to this date, academic and policy literature does not have a well-developed model to analyze these critical issues.

This study aims to fill that void in the literature and provides a theoretical model of MMLR. We first characterize the MMLR's "announcement effect," where asset prices increase immediately following the announcement of future liquidity provision, even without any actual asset purchases. We also show that the central bank can expect to buy less ex-post by committing to buy more ex-ante. Here, more audacious actions paradoxically lead to more conservative outcomes, which is beneficial for a central bank that wishes to constrain its balance sheet expansion and money supply while attempting to prevent disruptions in the financial market. We then present the optimal policy and discuss potential fragilities in implementing this policy due to multiple self-fulfilling equilibria. The multiplicity may arise if the central bank does not intervene with sufficient aggression or if market participants have doubts about its commitment. Lastly, we examine distortions in private

[2] The implementation of OMT involved various challenges with concerns being raised about the program's legality. See *German government defends ECB bonds after first day in court*, available at https://www.dw.com/en/german-government-defends-ecb-bonds-after-first-day-in-court/a-16875177

[3] See the interview with Paul Tucker for the debate regarding the MMLR, available at https://www.moneyandbanking.com/commentary/2015/3/4/interview-with-paul-mw-tucker.

incentives that may arise if the MMLR access becomes permanently available.

Our three-period model features the interactions among long-term investors (non-banks including insurance companies, mutual funds, or pension funds, referred to as “insiders”), liquidity providers (dealers, referred to as “outsiders”), and a central bank. At $t = 0$, insiders receive a liquidity shock requiring a cash injection, which they meet by selling their assets to market-making outsiders (Duffie, 2010). The amount of assets insiders need to liquidate depends on market liquidity and the price bid by outsiders. Specifically, since more assets need to be sold to generate the required cash with a lower liquidation price, the scale of $t = 0$ liquidations increases when outsiders bid a lower purchase price.

While providing liquidity to make markets, outsiders are not efficient users of the assets. Hence, they acquire the assets with the intention of selling back to more efficient buyers at $t = 1$, rather than holding them until maturity at $t = 2$. Consequently, with market competition, their willingness to pay at $t = 0$ depends on the expected future price at $t = 1$ for them to break even in expectation. Here, future market liquidity affects the cost of liquidity provision with immediacy by dealers at $t = 0$.

Insiders receive some funds later at $t = 1$, the amount of which is randomly distributed.[4] Being the efficient users of the assets, they use this cash to buy back the assets from outsiders, where the price would depend on the funds available to insiders. Specifically, for a given amount of assets held by outsiders, the price would be equal to the fundamental value when insiders have enough cash to buy the entire inventory of assets from the outsiders at that price. However, the price would fall below the fundamental value when there is insufficient cash available in the market, resulting in cash-in-the-market pricing (Allen and Gale, 1994, 1998). Hence, future asset prices are also affected by the scale of outsiders’ inventory, with the inventory becoming larger if insiders have sold more assets at $t = 0$.

Given these features, an interrelation arises between current and future liquidity. Outsiders’ expectation about the future price at $t = 1$ influences their willingness to pay at $t = 0$. This subsequently affects the scale of early liquidations at $t = 0$. At the same time, the scale of liquidations

[4] Alternatively, we can interpret this as a random arrival of capital that can run the asset efficiently but is slow-moving (Mitchell et al., 2007; Duffie, 2010; Acharya et al., 2013).

at $t = 0$ affects the future price at $t = 1$ due to potential cash-in-the-market pricing. With this interdependence, the asset price in equilibrium comprises a fixed point.

Note that a negative spiral can arise, exacerbating the liquidity dry-up if outsiders anticipate the future price of their inventories to be low. The negative prospects for the future limit their willingness to provide liquidity with immediacy at $t = 0$, leading to more fire-sales by insiders. Larger liquidations subsequently increase outsiders' inventory and depress future prices, further constraining their market-making incentives to cause a sharp decrease in asset prices and disorderly liquidations. To prevent such disruptions, the central bank can step in as a market maker of last resort by introducing a liquidity backstop through an asset purchasing facility.

Specifically, at $t = 0$, the central bank announces a capacity of the facility denoted as L, where it promises to inject up to L units of liquidity to purchase assets from outsiders at $t = 1$. This intervention can result in a strong announcement effect that instantly supports the price at $t = 0$, restraining disorderly liquidations. The effect comes from two channels that reflect the interrelation between $t = 0$ and $t = 1$ liquidity. First, the intervention directly affects the future asset price by increasing total cash available in the market at $t = 1$. This prospect of higher future prices immediately increases outsiders' willingness to pay at $t = 0$, resulting in reduced early liquidations. Furthermore, an indirect effect arises, amplifying the direct effect. Smaller liquidations at $t = 0$, in turn, reduce the scale of outsiders' inventory and improve their prospects of selling them at a better price at $t = 1$. This again promotes their market-making incentives at $t = 0$, generating a positive spiral (see Figure 1). Hence, the scale of the announcement effect depends on the scales of these direct and indirect effects.

Interestingly, we show that when the facility capacity L is sufficiently large, the central bank can reduce the expected usage of the facility at $t = 1$ by announcing a larger capacity at $t = 0$. This means that the central bank can anticipate buying less ex-post by demonstrating a stronger willingness to do more ex-ante. This outcome is advantageous for a central bank that needs to limit its money supply or balance sheet expansion while ensuring financial stability. Note that, as a last resort, the central bank does not need to take any action if there is enough insider liquidity in the market to maintain the asset price at the fundamental value. The likelihood of this occurring at

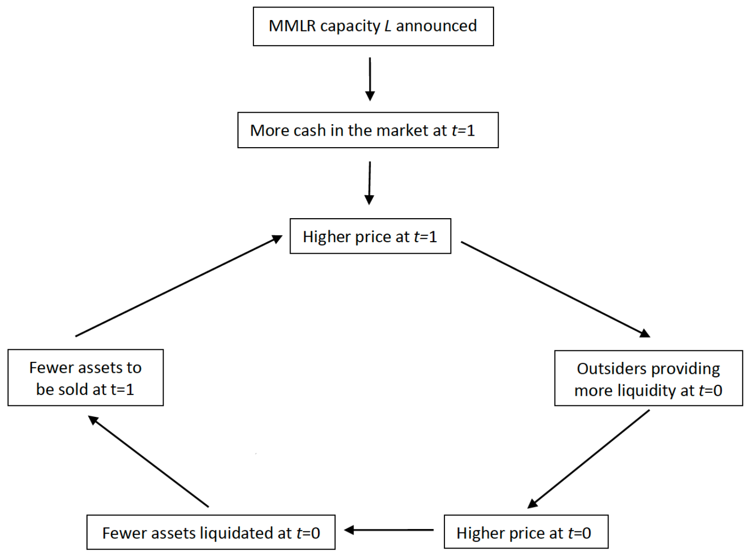

Figure 1: MMLR announcement and the feedback effects.

$t = 1$ increases if fewer assets are sold at $t = 0$. This is because, with fewer assets held by outsiders, it becomes more likely that the newly arriving insider liquidity can sufficiently support the $t = 1$ price at the fundamental value, even without any (or with a small) injection of public liquidity. As the central bank's commitment L increases, the expected future price therefore rises. This, in turn, boosts the current asset price and limits disorderly sales by insiders. When the facility capacity L exceeds a certain threshold, further expansion reduces the number of $t = 1$ states that would require public liquidity injections, thus decreasing the usage of the facility. In this case, we can observe a negative relation between the initial commitment and the expected usage of the facility.[5] This is exactly what would constitute a successful facility, as mentioned in the opening quote by Cecchetti and Tucker 2021.

Despite this beneficial feature, we argue that the MMLR intervention can have certain drawbacks and may not be suitable for all central banks. While the central bank can economize on

[5] In her speech *Liquidity Shocks: Lessons Learned from the Global Financial Crisis and the Pandemic* delivered on August 11, 2021, Lorie Logan made a similar point: "If intermediaries or end investors are confident that liquidity will be available in the future, either in the form of funding or asset purchases, they may perceive market-making and investing as less risky today—restoring the flow of transactions before any central bank operations are conducted. ... To the extent that announcements of central bank actions can reduce that liquidity demand and encourage a return to normal investing and market-making activity, they can significantly improve conditions even with little or no actual activity."

the expected usage of the facility owing to the positive spiral amplifying the announcement effect, this exact feedback effect may result in multiple self-fulfilling equilibria. In the "good" equilibrium, outsiders actively make markets at $t = 0$ in anticipation of high future prices. This instantly calms markets, and the central bank ends up doing very little as intended. However, in the "bad" equilibrium, outsiders are somehow pessimistic about future prices, which constrains their market-making incentives at $t = 0$. This leads to substantial asset liquidations at $t = 0$, forcing the central bank to buy more assets at $t = 1$. Yet, the asset price at $t = 1$ remains low with significant outsider inventories being sold, making the pessimistic belief self-fulfilling. This fragility indicates that the central bank may not achieve the intended outcome of restraining fire-sales and central bank purchases. Instead, the policy can still result in significant disorderly liquidations and public liquidity injections.

We first show that such multiple equilibria can arise if the central bank does not intervene with sufficient aggression, specifically when the facility capacity L is not large enough. This suggests that the central bank may sometimes need to adopt an *overly* aggressive strategy, such as promising to do "whatever it takes," in order to eliminate bad equilibria and avoid fragility, even if it is not the first-best option.[6] Moreover, we demonstrate that this fragility can arise if the central bank's commitment becomes an issue due to certain factors, such as time inconsistency or political pressures. For example, outsiders might doubt whether the central bank would indeed inject substantial liquidity ex-post if inflation pressures made a larger money supply more costly.[7] Hence, to avoid this fragility, central banks should intervene at a large enough scale, and market participants should have confidence that the central bank will honor its commitment.[8]

[6]When requesting the authority to extend credit to Fannie Mae and Freddie Mac with no explicit limit in July 2008, former U.S. Treasury Secretary Henry Paulson made a similar point: "If you've got a bazooka, and people know you've got it, you may not have to take it out. ... By increasing confidence, it will greatly reduce the likelihood it will ever be used." However, the U.S. Congress was reluctant to provide such an unprecedented blank check to one official, and the intervention was not effective in stabilizing financial markets.

[7]When announcing its emergency gilt-buying program in September 2022, which turned out to be less effective, the Bank of England indicated that the intervention was temporary and would unwind the purchased assets upon the program termination to avoid conflicting with its effort to constrain inflation. Besides the central bank's balance sheet constraints driven by its monetary policy objectives, the commitment problem may also stem from its reluctance to get exposed to certain types of credit risk or political concerns that can arise between central banks and governments.

[8]In discussing the Fed's response to the pandemic, Brunnermeier and Krishnamurthy (2020) note that "(o)ur conjecture is that the Fed's announcement has been viewed by the market as a "whatever it takes" moment. That is, the commitment to act aggressively in the high yield bond market has been taken as a signal of the Fed's willingness to defuse future episodes of financial instability in the broad credit market. This commitment has removed a bad equilibrium and reduced market tail risk. If our conjecture is correct, then the Fed does not currently need to make

The MMLR intervention could also distort private incentives if it becomes part of central banks' permanent toolkit. In our extension, we show that the availability of public liquidity can crowd out private liquidity because the intervention makes hoarding private liquidity less attractive. This nullifies the benefit of the public backstop and, in that case, MMLR would simply replace private liquidity. The consequent decrease in private liquidity would force the central bank to use the facility more often while little promoting overall market liquidity. In sum, our results suggest that a priori, the MMLR option should avail itself of access only during exceptional systemic events. Simultaneously, the central bank should be prepared to act with sufficient aggression if it chooses to utilize the MMLR.

The paper is related to the vast literature on central bank interventions during liquidity crises that date back to Thornton (1982) and Bagehot (1873). The literature has primarily focused on the LoLR role of central banks in the traditional bank-based system, which provides a backstop for funding liquidity to contain bank runs.[9] The modern financial system, on the other hand, is more market-based with the substantial growth of non-bank intermediaries, where dealers' provision of market liquidity in the presence of fire-sales is of central importance for financial stability (Brunnermeier and Pedersen, 2009; Tucker, 2009; Duffie, 2010). More recent studies, particularly following the Global Financial Crisis, analyzed the role of public interventions on market liquidity and financial stability (e.g., Acharya and Yorulmazer 2007, Acharya et al. 2010, Diamond and Rajan 2011, Acharya et al. 2012, and Stein 2012). This paper differs from them in that it theoretically formalizes the mechanism of the newly introduced MMLR operation that provides a liquidity backstop for private dealers. While the interventions examined in the prior studies require an actual liquidity injection through direct lending or asset purchasing, the MMLR facility, like the LoLR, may not be used after all if the public backstop successfully reinstates market liquidity (Tucker 2009, Mehrling 2010). To the best of our knowledge, this paper is the only article that provides a theoretical framework to delve into the efficacy of this new tool.

The MMLR interventions attracted much attention recently with their remarkable success during

good on its promise and activate the corporate bond purchase program at this point in time. The important aspect of the Fed's announcements has been the signal of its willingness to act if dislocations arise, and reinforcing this commitment is all that is needed at present."

[9]See, e.g., Bordo (1990), Santos (2006) and Ennis (2016) for the surveys of prior studies.

the Covid-19 pandemic. A number of studies empirically document how they instantly restored liquidity upon their introduction when the dealers' market-making capabilities were constrained (see, e.g., Brunnermeier and Krishnamurthy 2020; Boyarchenko et al. 2021; Haddad et al. 2021; Kargar et al. 2021; Ma et al. 2021; O'Hara and Zhou 2021; Vissing-Jorgensen 2021). However, more recent interventions by the Bank of England and the ECB in late 2022 were much less effective than the precedents, questioning their robustness. Our model provides important policy implications by presenting fragilities in implementing the MMLR policy to reconcile these conflicting outcomes, also discussing possible distortions to arise should this unconventional measure be included in the central bank's permanent toolkit.

More broadly, this paper is related to the literature on the effect that intermediary frictions impose on market liquidity (see, e.g., Gromb and Vayanos (2002); Brunnermeier and Pedersen (2009); Duffie (2010)), focusing on the cost of dealers' immediacy provision (see, e.g., Grossman and Miller 1988; Bao et al. 2018; Bessembinder et al. 2018; Goldberg and Nozawa 2021; He et al. 2022; Choi et al. 2023). We contribute to this literature by examining the role of public backstops and their downsides, shedding light on the interplay between private and public liquidity.

The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 presents the positive results. Section 4 discusses the optimal policy and its potential fragility. Section 5 discusses other policy options and extends the baseline model to examine private incentive distortions. Section 6 concludes.

2 Model

In this section, we introduce the model, its agents and timeline, and define the equilibrium.

2.1 Agents and asset markets

We consider a model with three dates: $t = 0, 1, 2$. The economy has insiders (long-term investors experiencing liquidity shocks), outsiders (dealers providing immediacy), and a central bank.[10] There

[10] Insiders include non-bank institutions such as pension funds, insurance companies, mutual funds or asset managers. We specifically focus on non-banks since the paper analyzes the MMLR role of the central bank, whereas banks already have access to the LoLR facilities provided by central banks.

is a continuum of insiders with measure 1, each endowed with long-term assets that mature at $t = 2$ and generate a return of R per unit when run by insiders. Insiders get a liquidity shock at $t = 0$ and need to sell some of their assets to generate the funds needed. They later receive some funds at $t = 1$ that they can use to buy back (some of) the assets sold at $t = 0$.

Outsiders do not have any projects to invest in but have deep pockets to purchase assets when they are up for sale.[11] However, outsiders are not the efficient users of these assets, that is, they can generate only $R - \Delta$ per unit when they run and hold the asset until maturity.[12] Hence, insiders value the asset higher than outsiders, and outsiders acquire the assets as temporary market makers, with an intention to sell back to insiders afterward. We assume that outsiders are risk-neutral with discount rate equal 1.

Insiders are hit by a liquidity shock at $t = 0$, which forces them to sell some of their assets. We assume that this shock is system-wide so that there is no financial capacity within the insiders to acquire the assets, and thus the assets need to be sold to outsiders at $t = 0$. The amount of assets sold by the insiders, denoted by α, depends on their liquidation price P_0. We assume (i) $\alpha'(P_0) \leq 0$, that is, when the price is lower, more assets need to be sold, and (ii) $\alpha''(P_0) \geq 0$, that is, sales increase in a weakly convex fashion as the price P_0 decreases.[13] Since outsiders are not efficient in running the assets, their willingness to pay at $t = 0$ depends on the price they anticipate to sell the assets at $t = 1$. Note that outsiders prefer to sell the asset back to an insider at $t = 1$ for any price greater than $R - \Delta$. We assume that the asset market at $t = 0$ is competitive, where outsiders break even in equilibrium.

Insiders receive new funds at $t = 1$, which we denote as I_1 and is randomly distributed with a continuous probability density function $f(.)$ (and cumulative distribution function $F(.)$) over the

[11] We assume that due to financial frictions insiders cannot readily borrow from outsiders to meet their liquidity demands.

[12] A positive Δ, which we assume to be exogenous and constant for simplicity, provides an incentive for outsiders to sell their inventory assets before maturity. This cost can be attributed to various factors, such as the lack of specialities or inventory costs incurred by market makers. See, for example, Stoll (1978), Amihud and Mendelson (1980), and Grossman and Miller (1988) for models featuring dealers' costs associated with providing liquidity with immediacy.

[13] This would be the case if, e.g., insiders need to raise c at $t = 0$ by liquidating the assets at the price P_0, which implies $\alpha = \frac{c}{P_0}$ satisfying $\alpha'(P_0) \leq 0$ and $\alpha''(P_0) \geq 0$. Negative relation between α and P_0 can also arise from, e.g., fire sales (Shleifer and Vishny, 1992) and cash-in-the-market pricing (Allen and Gale, 1994, 1998). Furthermore, a number of empirical studies (e.g., Chen et al., 2010; Goldstein et al., 2017; Falato et al., 2021; Ma et al., 2021) document that decreased asset price due to illiquidity induces further mutual fund redemptions. In such cases, the asset managers need to raise more funds as the price declines, which amplifies fire-sales and results in the convexity.

interval $[0, \bar{I}]$ as of $t = 0$.[14] We assume this to be uniformly distributed for expositional simplicity. As the efficient users of the assets, they use their cash I_1 to buy back the assets they sold to outsiders. The price at $t = 1$, denoted as P_1, depends on the amount of cash insiders have, following cash-in-the-market pricing (Allen and Gale, 1994, 1998). That is, given the amount of assets held by outsiders, the asset price would equal the fundamental value R when insiders have sufficient cash inflows to buy the entire outsider inventories at that price, but falls below it with limited cash available in the market. We elaborate on this below.

2.2 Central bank intervention

Limited liquidity at $t = 1$ would add downward pressure on the asset price P_1. The prospect of low future prices in turn diminishes outsiders' willingness to provide liquidity at $t = 0$. This leads to a low price P_0 and more fire-sales at $t = 0$, which further depresses future prices. To prevent such a negative spiral, the central bank can step in as a market maker of last resort by providing a liquidity backstop.

Suppose that the central bank introduces a facility with capacity L, that is, it promises to inject up to L units of liquidity to purchase assets at $t = 1$.[15] When the central bank injects L at $t = 1$, the total liquidity available for asset purchases is $I_1 + L$. Note that outsiders would sell their inventory assets at $t = 1$ as long as $P_1 > R - \Delta$. Given the size of outsiders' inventory α, the market-clearing price P_1 depends on the amount of available liquidity, and we have:

- For $I_1 + L \geq \alpha R$, there is enough liquidity in the market to sustain the price at the fundamental value R for all assets.
- For $\alpha(R - \Delta) \leq I_1 + L < \alpha R$, the price of the asset is determined by the available liquidity in the market, that is, $P_1 = \frac{I_1 + L}{\alpha}$, which we refer to as cash-in-the-market pricing (CIMP).
- For $I_1 + L < \alpha(R - \Delta)$, we have $P_1 = R - \Delta$ and outsiders would not sell the asset since they can generate $R - \Delta$ by holding the asset until maturity.

[14] This could also be interpreted as an arrival of new insiders with slow-moving capital (Mitchell et al., 2007; Duffie, 2010; Acharya et al., 2013). We assume $\bar{I}$ to be not very small.

[15] The optimal MMLR capacity, of course, should be chosen based on specific objectives of central banks. For now, we treat the MMLR capacity as given, discussing the optimal choice in Section 4.

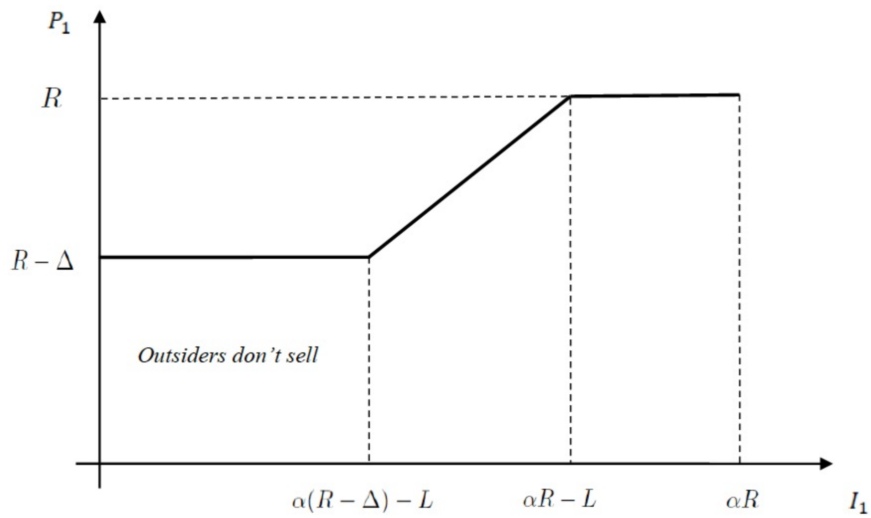

Figure 2: Price P_1 as a function of insider capital I_1.

Hence, the asset price P_1 at $t=1$ can be written as:

$$P_1 = \begin{cases} R & \text{for} \quad I_1 + L \geq \alpha R \\ \frac{I_1+L}{\alpha} & \text{for} \quad \alpha(R-\Delta) \leq I_1 + L < \alpha R \\ R-\Delta & \text{for} \quad I_1 + L < \alpha(R-\Delta) \end{cases}, \tag{1}$$

where Figure 2 provides an example illustrating P_1 as a function of I_1, given the facility capacity L and outsiders' inventory α.

2.3 Timeline and equilibrium

The timeline of the model is given as follows. At $t=0$, the central bank announces the capacity of the facility L. Outsiders then choose P_0, the price they are willing to pay for the asset, which determines α. At $t=1$, I_1 realizes and the central bank injects additional liquidity to acquire the assets from outsiders. At $t=2$, the return from the asset is realized.

Next, we define the equilibrium of the model. Given that the asset market at $t=0$ is competitive and outsiders are risk-neutral with discount rate equal 1, outsiders' willingness to pay at $t=0$ equals $E[P_1]$. Note that P_0 is the only choice variable given the capacity L of the MMLR facility. Here, P_0

is a rational expectations equilibrium if it satisfies

$$P_0 = E[P_1] \tag{2}$$

where P_1, given in (1), is a function of α and L. Since α is a function of P_0, this can be written as $P_0 = E[P_1(\alpha(P_0), L)]$, where the equilibrium P_0 is the fixed point of this equation.

3 Positive results

In this section, we examine the effect of the capacity of the central bank facility on the equilibrium price P_0 and the expected usage of the facility. We start by characterizing P_0 and its response to changes in the size of the facility L, where changes in P_0 lead to changes in asset sales α at $t = 0$.[16] The actual liquidity injection by the central bank at $t = 1$, denoted as $\tilde{L}$, depends on the amount of liquidity I_1 insiders have, as well as the amount of assets α that have been sold at $t = 0$. This is because, as a last resort, the central bank does not need to do anything if there arrives enough insider liquidity in the market to sustain the asset price at the fundamental value at $t = 1$. In other words, $\tilde{L}$ is a random variable as of $t = 0$ and the usage of the facility at $t = 1$ can be smaller than the facility's capacity L when private liquidity I_1 turns out to be large or outsider inventory α is small. We characterize the expected usage of the facility $E[\tilde{L}]$ and show that the expected usage can decrease in the size of the facility L when L is greater than a certain threshold. Hence, an aggressive central bank commitment can lead to fewer asset sales at $t = 0$ and, in turn, lower usage of the facility at $t = 1$. We finally discuss the possibility of multiple self-fulfilling equilibria in Section 3.3.

3.1 Price P_0 and the announcement effect

Next, we examine how the price P_0 responds as the capacity of the facility L increases, which we refer to as an "announcement effect".

Note that $P_0 = E[P_1(\alpha(P_0), L)]$ in equilibrium, and thus, we have

[16] As discussed in Section 3.3, multiple equilibria may exist. In Sections 3.1 and 3.2, where we analyze comparative statics, we momentarily assume that the central bank can choose a preferred equilibrium.

$$\frac{dP_0}{dL} = \frac{\partial E[P_1]}{\partial L} + \left[\frac{\partial E[P_1]}{\partial \alpha} \times \frac{\partial \alpha}{\partial P_0}\right] \times \frac{dP_0}{dL},$$

which gives us

$$\frac{dP_0}{dL} = \frac{\frac{\partial E[P_1]}{\partial L}}{1 - \left[\frac{\partial E[P_1]}{\partial \alpha} \times \frac{d\alpha}{dP_0}\right]}, \tag{3}$$

where we assume $\frac{\partial E[P_1]}{\partial \alpha} \times \frac{d\alpha}{dP_0} < 1$ to guarantee a stable fixed point. The numerator reflects the direct effect of the central bank facility on the asset price, whereas the denominator reflects a feedback effect that amplifies the direct effect, as illustrated in Figure 1. In particular, the expectation of higher future prices promotes outsiders' willingness to bid higher prices P_0 at $t = 0$, which reduces early liquidations α. Smaller α, in turn, improves outsiders' prospects to sell the assets they acquired at a high price P_1 at $t = 1$, which again increases P_0 to generate a positive spiral. The scale of the marginal announcement effect $\frac{dP_0}{dL}$ in equilibrium depends on these direct and indirect effects, that is, $\frac{\partial E[P_1]}{\partial L}$ and $\frac{\partial E[P_1]}{\partial \alpha}$.

Next, we characterize the expected price $E[P_1]$ as of $t = 0$. From equation (1) that characterizes P_1, we know that given outsiders' inventory α, the price P_1 can take three different cases depending on the available liquidity $L + I_1$ in the market at $t = 1$: (a) the lower bound $R - \Delta$ for low levels of liquidity; (b) CIMP given by $\frac{L+I_1}{\alpha}$ for intermediate levels of liquidity; and (c) the fundamental value R for high levels of liquidity. Hence, $E[P_1]$ will be the expected value out of these possible three cases, and depending on the facility capacity L we have:

(i) For $L \leq \alpha(R - \Delta) - \bar{I}$, we always have $P_1 = R - \Delta$ at $t = 1$ so that $E[P_1] = R - \Delta$;

(ii) For $\alpha(R - \Delta) - \bar{I} < L < \alpha R - \bar{I}$, P_1 can have the value $R - \Delta$ for low values of I_1 and also CIMP at $t = 1$ for large enough I_1;

(iii) For $\alpha R - \bar{I} < L < \alpha(R - \Delta)$, P_1 can take all three possible cases;

(iv) For $\alpha(R - \Delta) < L < \alpha R$, P_1 is given by CIPM for low values of I_1 or the fundamental value R for large I_1;

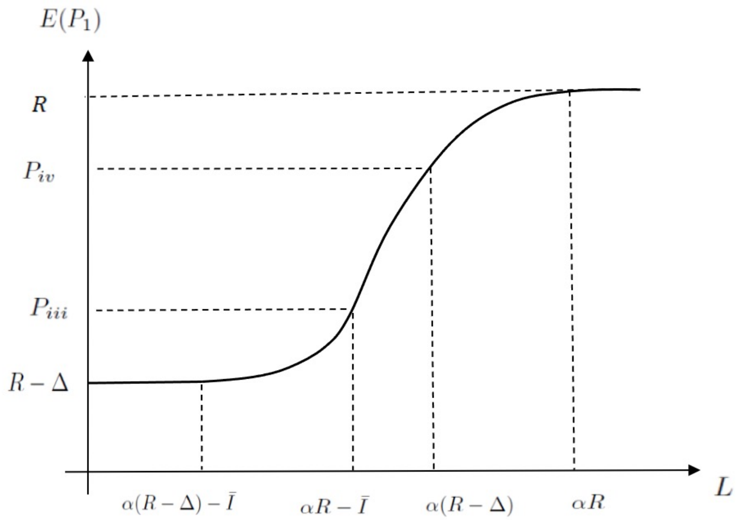

Figure 3: Figure illustrates the expected price $E(P_1)$ as a function of the capacity of the facility L for a uniform distribution for insider liquidity I_1.

(v) For a sufficiently large L with $L \geq \alpha R$, we always have $P_1 = R$ so that $E[P_1] = R$.

This gives us:

$$E[P_1] = \begin{cases} R-\Delta & \text{(i) if } L < \alpha(R-\Delta)-\overline{I} \\ (R-\Delta)F(\alpha(R-\Delta)-L) + \int_{\alpha(R-\Delta)-L}^{\overline{I}} \frac{(L+I_1)}{\alpha} f(I_1)dI_1 & \text{(ii) if } \alpha(R-\Delta)-\overline{I}<L<\alpha R-\overline{I} \\ (R-\Delta)F(\alpha(R-\Delta)-L) + \int_{\alpha(R-\Delta)-L}^{\alpha R-L} \frac{(L+I_1)}{\alpha} f(I_1)dI_1 + R\left[1-F(\alpha(R-L)\right] & \text{(iii) if } \alpha R-\overline{I}<L<\alpha(R-\Delta) \\ \int_0^{\alpha R-L} \frac{(L+I_1)}{\alpha} f(I_1)dI_1 + R\left[1-F(\alpha(R-L)\right] & \text{(iv) if } \alpha(R-\Delta)<L<\alpha R \\ R & \text{(v) if } L>\alpha R \end{cases}, \tag{4}$$

which is illustrated in Figure 3.

We now derive $\frac{\partial E[P_1]}{\partial L}$, the direct effect in equation (3). We know that $E[P_1]$ is constant in cases (i) and (v) so that $\frac{\partial E[P_1]}{\partial L} = 0$. For the other three intermediate cases, using the Leibniz integral rule, we obtain:

$$\frac{\partial E[P_1]}{\partial L} = \begin{cases} \frac{1}{\alpha}\left[1 - F(\alpha(R-\Delta) - L)\right] & \text{(ii) if } \alpha(R-\Delta) - \overline{I} < L < \alpha R - \overline{I} \\ \frac{1}{\alpha}\left[F(\alpha R - L) - F(\alpha(R-\Delta) - L)\right] & \text{(iii) if } \alpha R - \overline{I} < L < \alpha(R-\Delta), \\ \frac{1}{\alpha}\left[F(\alpha R - L)\right] & \text{(iv) if } \alpha(R-\Delta) < L < \alpha R \end{cases} \tag{5}$$

which is continuous and strictly positive. Hence, an increase in the capacity of the facility directly increases the expected price $E[P_1]$ with more cash in the market.

We next derive $\frac{\partial E[P_1]}{\partial \alpha}$, which determines the feedback effect in equation (3). Again, $E[P_1]$ is constant in cases (i) and (v) so that $\frac{\partial E[P_1]}{\partial \alpha} = 0$. For the other three cases, using the Leibniz integral rule, we obtain:

$$\frac{\partial E[P_1]}{\partial \alpha} = \begin{cases} -\int_{\alpha(R-\Delta)-L}^{\overline{I}} \frac{(L+I_1)}{\alpha^2} f(I_1) dI_1 & \text{(ii) if } \alpha(R-\Delta) - \overline{I} < L < \alpha R - \overline{I} \\ -\int_{\alpha(R-\Delta)-L}^{\alpha R - L} \frac{(L+I_1)}{\alpha^2} f(I_1) dI_1 & \text{(iii) if } \alpha R - \overline{I} < L < \alpha(R-\Delta), \\ -\int_{0}^{\alpha R - L} \frac{(L+I_1)}{\alpha^2} f(I_1) dI_1 & \text{(iv) if } \alpha(R-\Delta) < L < \alpha R \end{cases} \tag{6}$$

which is continuous and strictly negative. Hence, a decrease in asset liquidations at $t = 0$ promotes the future asset price at $t = 1$ with a smaller inventory of assets to sell by outsiders. Therefore, we have $\frac{\partial E[P_1]}{\partial L} \geq 0$, $\frac{\partial E[P_1]}{\partial \alpha} \leq 0$, and $\frac{d\alpha}{\partial P_0} < 0$. These in (3) give us our first main result $\frac{dP_0}{dL} \geq 0$, that is, as the capacity L of the central bank facility increases, the price P_0 of assets at $t = 0$ increases resulting in fewer sales α at $t = 0$.

Proposition 1. *We have $\frac{dP_0}{dL} \geq 0$ and $\frac{d\alpha}{dL} \leq 0$.*

In sum, a possible intervention by the central bank would directly increase the expected future price with more cash in the market. This, in turn, promotes outsiders' liquidity provision at $t = 0$ and thus increases P_0. Furthermore, the indirect effect amplifies this direct effect. That is, a higher P_0 results in fewer asset liquidations α at $t = 0$, and with fewer assets purchased by outsiders at $t = 0$, fewer assets will be sold at $t = 1$ resulting in a further increase in $E[P_1]$. This again increases P_0 and the subsequent feedback amplifies the announcement effect.

Examining how the marginal effect changes as the capacity L increases, we have the following result.

Corollary 1. *$\frac{dP_0}{dL}$ is continuous and: (i) 0 if $L < \alpha(R-\Delta) - \overline{I}$; (ii) increasing in L if $\alpha(R-\Delta) - \overline{I} <$*

$L < \alpha R - \bar{I}$; (iii) constant if $\alpha R - \bar{I} < L < \alpha(R - \Delta)$; (iv) decreasing in L if $\alpha(R - \Delta) < L < \alpha R$; (v) 0 if $L > \alpha R$.

That is, the marginal effect $\frac{dP_0}{dL}$ is not very strong when $P_0 = E[P_1]$ is near its lower bound $R - \Delta$, but becomes more significant as the capacity L increases to promote the liquidation price. As the capacity further expands, the incremental effect starts to decline since the demand for market liquidity gets saturated to bring the price near its fundamental value R.

3.2 Usage of the facility

In this section, we analyze the usage of the facility at $t = 1$, denoted as $\tilde{L}$. Focusing on how the expected usage (and thus central bank balance sheet) responds to an increase in the facility capacity $\frac{dE[\tilde{L}]}{dL}$, we argue that rather surprisingly, the central bank can reduce the expected usage of the facility by announcing a larger capacity ex ante if the announcement effect is sufficiently strong.

Next, we characterize $\frac{dE[\tilde{L}]}{dL}$. Recall that at $t = 0$, the central bank announces to use up to L at $t = 1$ to purchase assets through its facility. First, note that from (1), we have $P_1 < R$ with probability 1 when the capacity of the facility is small with $L < \alpha R - \bar{I}$. In that case, the central bank would always have to intervene up to its full capacity at $t = 1$ regardless of I_1. Therefore, we simply have $E[\tilde{L}] = L$ and $\frac{dE[\tilde{L}]}{dL} = 1$, where an increase in the facility capacity is always associated with an increase in the expected usage.

When L is sufficiently large with $L > \alpha R$, we have $P_1 = R$ with probability 1 from (1). In that case, there is no unmet demand for liquidity and increasing the capacity L will not have any effect on the usage of the facility, that is, $\frac{dE[\tilde{L}]}{dL} = 0$.

With an intermediate capacity such that $\alpha R - \bar{I} < L < \alpha R$, the actual usage of the facility depends on the availability of insider liquidity I_1. Specifically, for $I_1 \geq \alpha R$, insiders have enough cash to pay R for all liquidated assets at $t = 1$ and the facility is not used at all, that is, $\tilde{L} = 0$. For $\alpha R - L \leq I_1 < \alpha R$, the price of the asset is R, where the facility is only partially used with $\tilde{L} = \alpha R - I_1$. For $I_1 < \alpha R - L$, the facility is fully utilized with $\tilde{L} = L$ but, even then, the price of

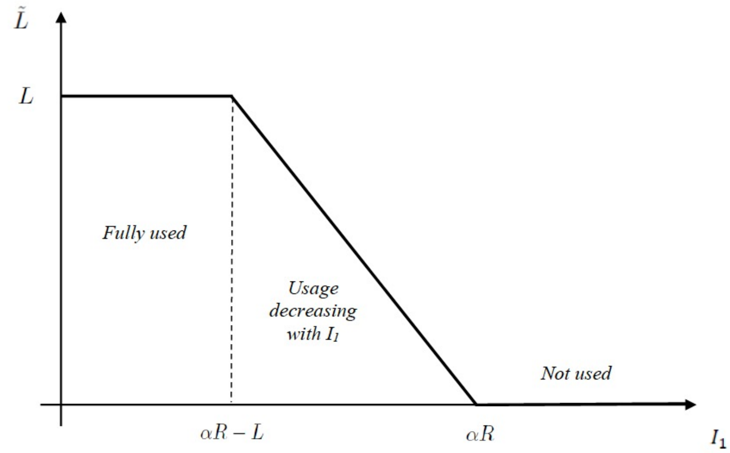

Figure 4: Usage of the facility as a function of insider liquidity I_1.

the asset cannot be sustained at R. This gives us:

$$\tilde{L} = \begin{cases} 0 & \text{for} \quad I_1 > \alpha R \\ \alpha R - I_1 & \text{for} \quad \alpha R - L \leq I_1 < \alpha R \\ L & \text{for} \quad I_1 < \alpha R - L \end{cases} \cdot$$

Figure 4 illustrates the usage of facility $\tilde{L}$ as a function of I_1 in this case.

Therefore, we can characterize the expected usage of the facility when $\alpha R - \overline{I} < L < \alpha R$ as follows:

$$E[\tilde{L}] = \int_0^{\alpha R - L} L f(I_1) dI_1 + \int_{\alpha R - L}^{\min\{\alpha R, \overline{I}\}} (\alpha R - I_1) f(I_1) dI_1 + \int_{\min\{\alpha R, \overline{I}\}}^{\overline{I}} 0 \times f(I_1) dI_1.$$

Note that the capacity of the facility has a direct effect through a change in L and an indirect effect through a change in α. Using the Leibniz integral rule, we obtain:

$$\frac{dE[\tilde{L}]}{dL} = \underbrace{F(\alpha R - L)}_{>0,\text{ greater usage}} + \underbrace{\frac{d\alpha}{dL} R \left[F\left(\min\{\alpha R, \overline{I}\}\right) - F(\alpha R - L) \right]}_{<0,\text{ less usage}}. \tag{7}$$

The first term is positive, reflecting the greater amount of liquidity injection required in the states with limited insider liquidity. Having announced a larger L, the central bank would need to inject additional liquidity in the future states with $P_1 < R$, which arises with probability $F(\alpha R - L)$.

The second term is negative indicating less injection being required in certain future states due to smaller outsider inventory α. Since the prospect of more aggressive interventions results in higher $E[P_1]$, this, in turn, increases P_0 and decreases α. This implies $\frac{d\alpha}{dL} = \frac{d\alpha}{dP_0} \times \frac{dP_0}{dL} \leq 0$. With fewer assets being sold at $t = 1$, a smaller amount of public liquidity injection at $t = 1$ can achieve $P_1 = R$. That is, the usage of the facility declines by $|\frac{d\alpha}{dL}|$ when having I_1 with $\alpha R - L < I_1 < \min\{\alpha R, \overline{I}\}$.

Note that an increase in L reduces the likelihood of states requiring more public liquidity injection (i.e., $F(\alpha R - L)$). At the same time, it increases the likelihood of states requiring less injection (i.e., $F\left(\min\{\alpha R, \overline{I}\}\right) - F(\alpha R - L)$). These two effects contribute to a decline in the expected usage $E[\tilde{L}]$ when announcing a larger capacity.

We can summarize $\frac{dE[\tilde{L}]}{dL}$ as:

$$\frac{dE[\tilde{L}]}{dL} = \begin{cases} 1 & \text{for } L < \alpha R - \overline{I} \\ F(\alpha R - L) + R\frac{d\alpha}{dL}\left[F\left(\min\{\alpha R, \overline{I}\}\right) - F(\alpha R - L)\right] & \text{for } \alpha R - \overline{I} < L < \alpha R. \\ 0 & \text{for } L > \alpha R \end{cases} \tag{8}$$

Here, $\frac{dE[\tilde{L}]}{dL}$ is continuous in L, and thus is positive with small enough L. For larger L, the expected usage of the facility can decrease in the capacity of the facility if the second negative term in (7) dominates the first positive term. That is, for $\alpha R - \overline{I} < L < \alpha R$, we have $\frac{dE[\tilde{L}]}{dL} < 0$ if

$$\frac{d\alpha}{dL}\ \left(= \frac{d\alpha}{dP_0} \times \frac{dP_0}{dL}\right) < -\frac{F(\alpha R - L)}{R\left[F\left(\min\{\alpha R, \overline{I}\}\right) - F(\alpha R - L)\right]}\ (< 0). \tag{9}$$

Note that the left hand side (LHS) above becomes smaller (more negative) with a larger $\frac{dP_0}{dL}$. That is, the expected usage of the facility can decrease in L if the announcement effect $\frac{dP_0}{dL}$ is significant enough to satisfy (9). Recall from Corollary 1 that $\frac{dP_0}{dL}$ is very small when P_0 is near its lower bound $R - \Delta$, but becomes larger as L increases. Since the right hand side (RHS) increases in L, this implies that while (9) would not hold for a sufficiently low liquidation price P_0, it would start to hold as the capacity L increases. Elaborating on this further, using (3) we can write (9) as:

$$\frac{d\alpha}{dP_0} \times \frac{\frac{\partial E[P_1]}{\partial L}}{1 - \frac{\partial E[P_1]}{\partial \alpha} \times \frac{d\alpha}{dP_0}} < -\frac{F(\alpha R - L)}{R\left[F\left(\min\{\alpha R, \overline{I}\}\right) - F(\alpha R - L)\right]},$$

that is,

$$\frac{d\alpha}{dP_0} < -\Big[\frac{\partial E[P_1]}{\partial L} \times \frac{R\left[F\left(\min\{\alpha R, \overline{I}\}\right) - F(\alpha R - L)\right]}{F(\alpha R - L)} - \frac{\partial E[P_1]}{\partial \alpha}\Big]^{-1}, \tag{10}$$

which gives the minimum slope $|\frac{d\alpha}{dP_0}|$ to have $\frac{dE[\tilde{L}]}{dL} < 0$.

Note that the RHS of (10) is continuous and monotonically increasing in L from $\frac{\partial E[P_1]}{\partial L}$ given in (5) and $\frac{\partial E[P_1]}{\partial \alpha}$ in (6). Hence, we obtain the following result.

Proposition 2. *The RHS of (10) is continuous and (weakly) increasing in L, with the minimum $\underline{\alpha}'$ and the maximum $\bar{\alpha}'$ for $\alpha R - \overline{I} < L < \alpha R$.*

Deriving exact conditions for (10) obviously requires specific assumptions about the functional form of $\alpha(P_0)$, while we have only assumed $\alpha'(P_0) \leq 0$ and $\alpha''(P_0) \geq 0$ so far. In our model, convexity of $\alpha(P)$ would amplify the feedback effect bolstering financial stability. However, it also adds complexity to the conditions necessary to satisfy (10). For expositional purposes, below, we examine a linear $\alpha(P_0)$ with $\alpha''(P_0) = 0$ to demonstrate the existence of a capacity threshold, where the sign of the inequality (10) flips across the threshold.

This reversal is straightforward from Proposition 2 since the RHS of (10) continuously increases in L, while $\frac{d\alpha}{dP_0}$ in the LHS becomes a constant. Therefore, given $\frac{d\alpha}{dP_0}$ with $\underline{\alpha}' < \frac{d\alpha}{dP_0} < \bar{\alpha}'$, there exists a threshold $L^{'}$ such that $\frac{dE(\tilde{L})}{dL} > 0$ when L is smaller than $L^{'}$, but $\frac{dE(\tilde{L})}{dL} < 0$ when L is greater than $L^{'}$. Similarly, Figure 5 illustrates a case with the threshold $L^{'}$ for non-linear $\alpha(P)$.

Corollary 2. *For any linear $\alpha(P)$ with $\underline{\alpha}' < \alpha'(P) < \bar{\alpha}'$, there exists an $L^{'}$ such that the expected usage of the facility declines as its capacity increases if (and only if) the capacity is larger than $L^{'}$.*

In sum, the central bank being aggressive in market making would reduce outsiders' concerns at $t = 0$ since they should be able to sell their inventories at a decent price P_1 to the insiders or the central bank at $t = 1$. This increases outsiders' willingness to act as temporary market makers

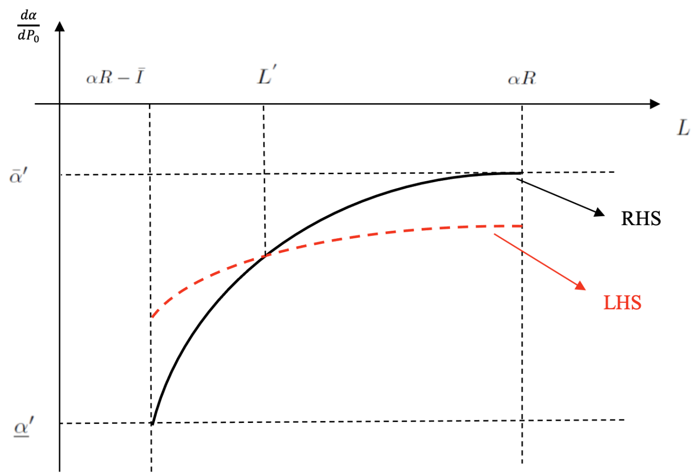

Figure 5: Figure illustrates a case where the expected usage of the facility starts to decline (i.e., $LHS < RHS$ satisfying (10)) as the capacity of the facility surpasses the threshold $L^{'}$.

and increase their bidding price P_0 at $t = 0$. The higher price at $t = 0$ leads to fewer sales α, and with fewer assets held by outsiders, it becomes more likely that insider liquidity on its own is sufficient to prop up the price P_1 to the fundamental value R at $t = 1$ without the (or with a small) assistance from the central bank. In this case, the central bank can expect to buy less by showing stronger willingness to buy more in case of necessity – seemingly audacious decisions can lead to more conservative outcomes.

3.3 Multiple self-fulfilling equilibria

To this point, we deliberately did not consider the potential for the existence of multiple equilibria to demonstrate our comparative statistics results. However, once the MMLR capacity L is determined, any $P_0 \in [R - \Delta, R]$ satisfying (2) can become an equilibrium outcome. In other words, multiple equilibria can emerge for a given capacity L, when there exist multiple fixed points satisfying $P_0 = E[P_1(\alpha(P_0), L)]$. We next discuss what factors contribute to such multiplicity.

We begin by examining how $E[P_1] \equiv E[P_1(\alpha(P_0), L)]$ changes with P_0. Technically, we have a fixed point when $E[P_1]$ as a function of P_0 intersects the 45-degree line (see Figure 6). We can write the derivative as

$$\frac{\partial E[P_1]}{\partial P_0} = \frac{\partial E[P_1]}{\partial \alpha} \times \frac{d\alpha}{dP_0}, \tag{11}$$

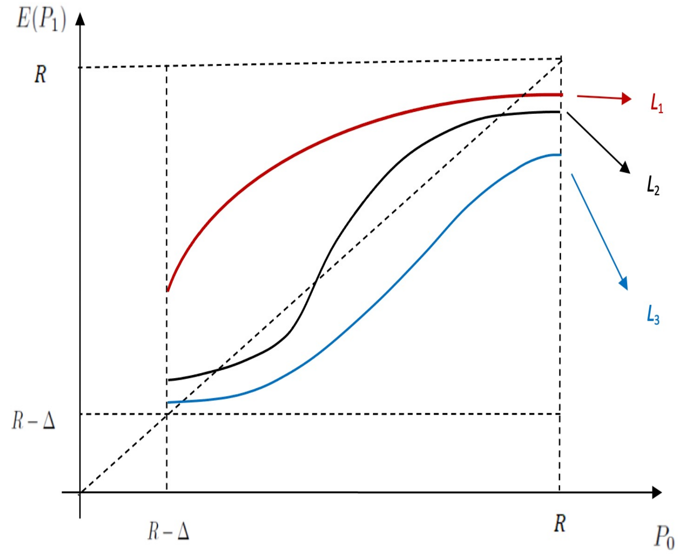

Figure 6: Illustration of potential multiple equilibria using three different facility capacities: large (L_1), moderate (L_2), and small (L_3) with $L_1 > L_2 > L_3$. A unique fixed point exists with L_1 or L_3, but multiple fixed points exist with L_2.

where $\frac{\partial E[P_1]}{\partial \alpha} \leq 0$ as we derived in equation (6) and $\frac{d\alpha}{dP_0} < 0$, hence $\frac{\partial E[P_1]}{\partial P_0} \geq 0$. Note that the RHS of (11) reflects the scale of the feedback effect in the characterization of the announcement effect in (3). Obviously, multiple fixed points would not exist if $\frac{\partial E[P_1]}{\partial P_0} < 1$ for all $P_0 \in [R-\Delta, R]$. Therefore, multiple equilibria can arise in the presence of a strong announcement effect with $\frac{\partial E[P_1]}{\partial P_0} > 1$; while the feedback effect enables the central bank to swiftly reinstate market liquidity with a limited usage of the facility, it can also pose a different challenge for the central bank.

We next discuss the shape of $E[P_1]$ as a function of P_0, i.e., the scale of the feedback effect $\frac{\partial E[P_1]}{\partial P_0}$, vis-a-vis the 45-degree line. Using $\alpha''(P_0) \geq 0$ and $\frac{\partial E[P_1]}{\partial \alpha}$ from equation (6), we get the following result.[17]

Proposition 3. *Given L, there exists $\tilde{P}_0(L)$ such that $\frac{\partial^2 E[P_1]}{\partial P_0^2} \leq 0$ for all $P_0 > \tilde{P}_0(L)$, and $\frac{\partial^2 E[P_1]}{\partial P_0^2} \geq 0$ for all $P_0 < \tilde{P}_0(L)$. The inflection point $\tilde{P}_0(L)$ weakly decreases in L.*

In other words, $E[P_1]$ increases in a concave fashion in P_0, except when $E[P_1]$ is close to the

[17]We ignore the third order effect $\alpha'''(P_0) \approx 0$ for expositional purposes. A weaker sufficient condition for our results would be for $\alpha''(P_0)$ to be monotone in P_0.

lower bound $R-\Delta$ where it becomes convex in P_0 as is displayed in Figure 6. The concave (convex) region becomes larger (smaller) as L increases, where, for sufficiently large L, the convex region disappears and $E[P_1]$ always increases in a concave way (case $L = L_1$ in Figure 6).

Recall from Corollary 1 that the announcement effect is not very strong when $E[P_1]$ is near its lower bound $R-\Delta$ or upper bound R, but becomes more significant as the price deviates from these limits. Intuitively, $E[P_1]$ being close to its lower bound $R-\Delta$ implies that P_1 would be $R-\Delta$ in most of the states at $t = 1$. Thus, most of the inventory assets are likely to be retained by outsiders with limited liquidity in the market. There, a marginal increase in P_0 that reduces α would increase *both* the asset price $\frac{L+I_1}{\alpha}$ under CIMP and the likelihood of CIMP, which results in the convexity when $E[P_1]$ is near $R-\Delta$. In contrast, when $E[P_1]$ is large, a further increase in P_0 with a smaller α would increase the likelihood of the $t = 1$ states where the market is saturated with liquidity such that $P_1 = R$, in which case a marginal decrease in α would have no additional impact on the asset price P_1. This results in the concavity when $E[P_1]$ is high enough. In addition, $\alpha(P_0)$ decreases in P_0 in a convex fashion, which makes $\frac{\partial E[P_1]}{\partial P_0}$ smaller for larger P_0. These give us the shape of $E[P_1]$ as a function of P_0 as characterized in Proposition 3. Since $E[P_1(\alpha(P_0), L)]$ increases in L, which converges to R for all P_0, we have the following results.

Corollary 3. *For a given facility capacity L^M, suppose there are multiple $P_0 \in [R-\Delta, R]$ satisfying $P_0 = E[P_1(\alpha(P_0), L^M)]$. Then there exists a facility capacity L^U such that $P_0 = E[P_1(\alpha(P_0), L)]$ has a unique fixed point if $L > L^U (> L^M)$.*

4 Optimal policy and fragility

In this section, we present the optimal policy of the central bank and analyze potential fragilities in its implementation due to multiple self-fulfilling equilibria and commitment problems that may arise.

4.1 Optimal policy

In this section, we characterize the optimal choice of the MMLR capacity. Here, our primary focus lies in illustrating the challenges central banks may face in achieving their intended outcome when making the MMLR intervention, rather than presenting an exhaustive characterization of the optimal policy itself. Given that the optimal decision hinges on the policy objectives that can be unique to individual central banks, we adopt a reduced-form objective function and focus on presenting the major trade-offs. For now, we temporarily set aside the possibility of multiple equilibria, addressing this aspect in the next subsection as we discuss fragility.

We consider a central bank facing the following trade-off when intervening as the MMLR. By providing the liquidity backstop, it aims to constrain disorderly liquidations (i.e., reducing α) because they can lead to misallocation costs and welfare losses. However, in achieving this, the central bank does not wish to use the facility excessively since it does not intend to "buy and hold" these assets, not being the most efficient runner of the assets.

Instead, the central bank inventory, if any, needs to be liquidated swiftly as the market recovers to minimize inefficiency. Here, the inefficiency cost increases with the amount of assets purchased by the central bank, as it would take longer to sell larger inventories. Besides the inventory cost, an increase in the facility usage also requires the central bank to expand its balance sheet and money supply, which may conflict with its monetary policy objectives such as curbing inflation.[18]

Therefore, we consider a central bank that (i) aims at limiting liquidations α at $t = 0$ but also (ii) attempts to economize its scale of interventions $E[\tilde{L}]$.[19] Hence, we assume that the central bank chooses the capacity of the facility L at $t = 0$ to minimize the loss function given by

$$\mathcal{L} = \gamma(\alpha) + E[\tilde{L}], \tag{12}$$

where γ increases in a weakly convex fashion in α such that more asset sales at $t = 0$ leads to a higher cost for the central bank. We assume the loss to be linear in $E[\tilde{L}]$ for simplicity.

[18]MMLR operations, therefore, differ from asset purchasing programs that do not intend to unwind the purchased assets quickly. When launching its emergency gilt-buying program in September 2022, the Bank of England made clear that it was a temporary measure and would unwind the purchased assets timely upon program termination, so as not to conflict with its effort to curb inflation.

[19]We obtain a similar result when alternatively considering a central bank that attempts to limit its asset acquisition, denoted as $E[\alpha_{CB}]$, where α_{CB} is the amount of assets the central bank purchases at $t = 1$, instead of $E[\tilde{L}]$. As indicated by equation (16) in Section 4.3, α_{CB} increases in $\tilde{L}$ for all I_1 so that a larger facility usage implies more asset acquisitions.

We can write the FOC for the interior solution as follows:

$$\frac{d\mathcal{L}}{dL} = \underbrace{\gamma^{'}(\alpha)\frac{d\alpha}{dL}}_{<0} + \underbrace{\frac{dE[\tilde{L}]}{dL}}_{>0 \text{ or } <0} . \tag{13}$$

The first term in the RHS is negative, as long as the market price responds to the liquidity injection, that is, $\frac{dP_0}{dL} > 0$. The central bank in this case can limit the costs of disorderly liquidations by increasing L.

The second term in the RHS can take both signs as discussed in Section 3.2. When the second term has a positive sign, an increase in the facility capacity would pose a trade-off. On the one hand, a larger capacity decreases asset liquidations at $t = 0$, which has a desirable effect for the central bank objective. On the other hand, a larger capacity increases the expected usage of the facility, which is costly. When $\frac{dE[\tilde{L}]}{dL} > 0$ for all L, the central bank will choose the optimal capacity L^* that balances the trade-off between the decrease in early liquidations, that is, $\gamma^{'}(\alpha)\frac{d\alpha}{dL}$, and the increase in the usage of the facility, that is, $\frac{dE[\tilde{L}]}{dL}$.

However, this trade-off disappears when we have $\frac{dE[\tilde{L}]}{dL} < 0$. In that case, a further expansion of the capacity of the facility is evidently desirable as it limits asset liquidations at $t = 0$ and, at the same time, decreases the expected usage of the facility, which *always* reduces the loss function $\mathcal{L}$.[20] It is obvious that any L with $\frac{dE[\tilde{L}]}{dL} < 0$ cannot be the optimal solution – the central bank should always increase its facility capacity in such cases.

This implies that the optimal capacity L^* may not have an interior solution. In Figure 5, for instance, we have $\frac{dE[\tilde{L}]}{dL} < 0$ for all $L^{'} < L < \alpha R$ so that $\frac{d\mathcal{L}}{dL}$ is also negative in that region and there is no trade-off arising from increasing the capacity L. Hence, the central bank could decrease $\mathcal{L}$ by increasing the capacity of the facility up to $L = \alpha R$, which can become the corner solution minimizing $\mathcal{L}$.[21] Note that $\frac{d\mathcal{L}}{dL} = 0$ for all $L > \alpha R$ since the market is fully saturated with liquidity and any

[20]Specifically, using $\frac{dE[\tilde{L}]}{dL}$ in equation (7), note that $\frac{d\mathcal{L}}{dL} < 0$ can be written as:

$$\frac{d\alpha}{dP_0} < -\Big[\frac{\partial E[P_1]}{\partial L} \times \frac{R\left[F\left(\min\{\alpha R, \bar{I}\}\right) - F\left(\alpha R - L\right) + \gamma'(\alpha)\right]}{F\left(\alpha R - L\right)} - \frac{\partial E[P_1]}{\partial \alpha}\Big]^{-1}, \tag{14}$$

where the *RHS* again increases continuously in L. This is a weaker condition than (10) since $\gamma'(\alpha) < 0$ so that $\frac{dE[\tilde{L}]}{dL} < 0$ becomes a sufficient condition for $\frac{d\mathcal{L}}{dL} < 0$.

[21]The optimality would hold if $\mathcal{L}$ with $L = \alpha R$ is smaller than the local minimum of $\mathcal{L}$ for $0 \leq L \leq L'$.

further increase in L will not have any additional effect, that is, we have $\frac{d\alpha}{dL} = 0$ and, thus, $\frac{dP_0}{dL} = 0$. Therefore, if the corner solution $L^* = \alpha R$, which would never leave any liquidity demand unmet, minimizes $\mathcal{L}$, any $L > \alpha R$ can also become an optimal capacity having the same $\mathcal{L}$. Nonetheless, some central banks deliberately declare that they would intervene in an *overly* aggressive way, or announce a "whatever it takes" policy. Next, we discuss how such strong forcefulness may make a difference in the presence of multiple equilibria by eliminating the potential bad equilibria.

4.2 Multiple equilibria and MMLR capacity

The optimality argument above implicitly assumed that the central bank could choose a preferred outcome with a smaller $\mathcal{L}$ in case there exist multiple equilibria given the choice of L^*. However, such arbitrary selection may not be possible, and the central bank could instead end up spending significantly without effectively constraining disorderly liquidations. We argue that this fragility can arise when the central bank does not intervene with sufficient capacity. Therefore, for the purpose of robustness, it may be better for the central bank to adopt an *overly* aggressive approach. We first examine the case with the corner solution as the optimal capacity, and then proceed to the case with an interior optimal capacity.

Corner solution – whatever it takes. We first discuss why the central bank may choose to be overly aggressive by announcing the "whatever it takes" policy. At the end of Section 4.1, we argued that for the case illustrated in Figure 5, for example, any $L \geq \alpha R$ could optimally saturate the demand for liquidity in the market to have $P_0 = R$ and become an optimal capacity. Still, given the potential fragility from multiple equilibria, the central bank may wish to announce a considerably large L to avoid the unintended sub-optimal outcomes arising in the bad equilibrium. To illustrate this point, Figure 7 compares two different capacities, L_H and L_M with $L_H \gg L_M > \alpha R$, where any fixed point P_0^* satisfying $P_0^* = E[P_1(P_0^*)]$ can become an equilibrium price.

Under the aggressive policy with the large capacity L_H, we only have a single fixed point $P_0 = R$, where we have the intended outcome with minimized loss $\mathcal{L}$. However, under the modest policy with the capacity L_M, we can additionally have $P_0^{'}$ and $P_0^{''}$ as equilibrium outcomes, where the central bank ends up with more asset liquidations α at $t = 0$ *and* a greater usage of the facility. In this

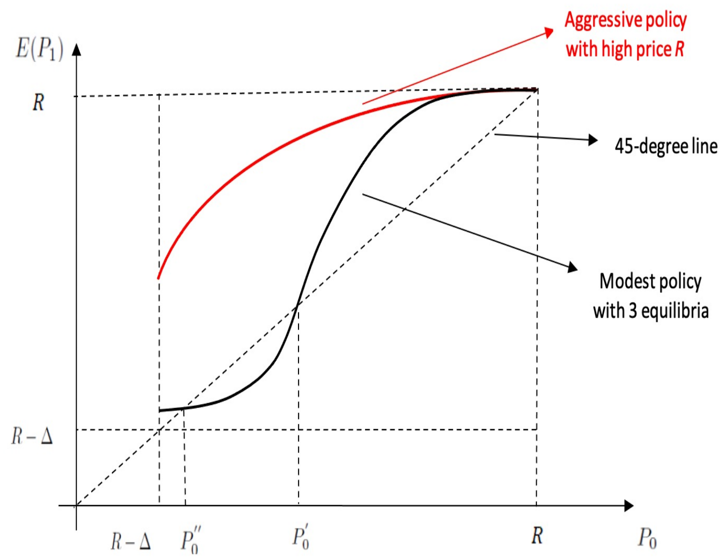

Figure 7: Figure illustrates how multiple equilibria can exist with a modest facility with $L = L_M$ whereas an aggressive policy such as "whatever it takes" (with $L = L_H$ where $L_H \gg L_M$) can eliminate multiple equilibria and achieve the good equilibrium.

case, the central bank should announce the aggressive policy with the large capacity L_H to influence off-the-equilibrium beliefs and eliminate the bad equilibria. Corollary 3 suggests that the central bank can indeed eliminate the bad equilibrium by adopting a large capacity.

Interior solution – overly aggressive announcement. Similar arguments can be made when we have an interior solution for the optimal capacity of the facility, that is, when $L^* < \alpha R$ with $P_0^* < R$. We revisit Figure 6 with three different capacity sizes to illustrate this issue. Suppose that the optimal capacity minimizing $\mathcal{L}$ is large with $L^* = L_1$. In that case, the facility would sufficiently support the market price P_1 at $t = 1$, and we have a unique equilibrium with a high P_0 and a low α. However, when the optimal capacity is modest with $L^* = L_2$, we have multiple equilibria that could be Pareto ranked – instead of the intended outcome with the high P_0 and a small loss $\mathcal{L}$, we may end up with worse outcomes resulting in a greater loss $\mathcal{L}$, where the central bank faces a lower P_0 and a higher α, as well as higher expected usage of the facility $E[\tilde{L}]$. When L^* is significantly small such that $L^* = L_3$, we again have a unique equilibrium but the MMLR policy does not seem very "effective" — the outcome is close to the "bad" equilibrium for the case of L_2

with large loss $\mathcal{L}$.

This raises an interesting discussion about what the central bank would do in the presence of multiple equilibria. Assuming the central bank can achieve an intended outcome, suppose we obtain $L^* = L_2$ as the optimal policy that minimizes $\mathcal{L}$. However, when such equilibrium selection is not possible, the central bank may end up with an unintended outcome having a larger $\mathcal{L}$. A cautious central bank may instead follow a robust strategy that would resemble a maximin strategy, where the central bank maximizes the payoff under the worst outcome. In that case, even though L_1 is not the optimal outcome from the FOC of the objective function, the central bank may still choose to implement it to avoid the potential bad equilibria arising with L_2 if the loss from the bad equilibrium is larger than the loss from implementing L_1. Hence, to prevent the fragility stemming from multiple equilibria, the central bank may opt for a second-best policy, acting *overly* aggressively by implementing $L > L_2$ that guarantees a unique equilibrium.

In sum, if the first-best policy features a not too large L^*, it is possible to have "good" and "bad" equilibria that are both self-fulfilling. In the good equilibrium, outsiders are willing to bid a high price anticipating they could later sell the acquired assets at a high price. Since outsiders provide more liquidity at $t = 0$, fewer fire-sales arise. The central bank may not need to intervene much at $t = 1$ since liquidity within the insiders would be sufficient to buy back these assets from outsiders, which results in a small loss $\mathcal{L}$ for the central bank. In contrast, in a bad equilibrium, outsiders in anticipation of low future prices bid a low price, causing substantial fire-sales at $t = 0$. The central bank then needs to inject high levels of liquidity at $t = 1$ in more number of states, yet the prices in those states can still be low. This is the bad self-fulfilling equilibrium with a large loss $\mathcal{L}$ for the central bank.

Importantly, the central bank can eliminate the bad equilibria by announcing a facility with a large capacity that can provide a substantial amount of liquidity in times of necessity. In that case, the central bank would *surely* be propping up the future price P_1, which encourages outsiders to provide liquidity at $t = 0$. The liquidity provision by outsiders at $t = 0$ limits liquidations α, which makes the pessimistic belief nonviable and eliminates the bad equilibria. On the contrary, the perspective of an intervention with a lesser capacity would sustain the pessimistic belief, making the bad equilibria self-fulfilling.

4.3 Commitment problems and multiple equilibria

We previously argued that central banks that are ready to intervene aggressively can eliminate multiple equilibria, thus achieving the intended outcome while getting to intervene less at the end. However, this is only possible when the central bank can indeed intervene at $t = 1$ as announced at $t = 0$, with no commitment problem arising from issues such as time inconsistency or political pressures, etc. Some are skeptical about whether these policies would always work as intended, questioning their robustness. Speaking of the OMT, Krugman says that "the ECB's efforts rely to an important extent on a bluff, in the sense that nobody knows what would happen if OMT were actually required." Recently, in announcing its gilt-purchase program in September 2022, the Bank of England needed to make clear that this would be a temporary measure with a set termination date since there was significant concern about inflationary pressures requiring a tighter money supply. Next, we analyze the fragility that may arise when the central bank has certain constraints in the ex-post implementation of its policy.

Whatever it takes, revisited. Let us revisit the two policies illustrated in Figure 7. The "whatever it takes" policy showed how the central bank's strong commitment can affect the outcome of the intervention. As we discussed in the previous section, the central bank can surely achieve the intended outcome with a small loss $\mathcal{L}$ having low α and low $E[\tilde{L}]$ by announcing the large capacity $L^* = L_H$, but may have a bad outcome with the modest capacity L_M due to the multiplicity of equilibria.

Now, suppose that the central bank has announced the large capacity L_H but, in fact, it cannot spend more than L_M at $t = 1$.[22] If outsiders believe in the central bank's commitment, then the only equilibrium would be the good equilibrium with $P_0 = R$ and a small α, where the central bank facing a small α does not need to intervene much at $t = 1$ — that is, $\tilde{L} \leq L_M$ with probability 1 and the "bluff" would work. However, if outsiders have doubts about the central bank's actual capability to intervene, they may choose P_0' or P_0'' instead, and the "bluff" can fail with a large α — since

[22] This practical limit can come directly from certain central bank objectives such as curbing inflation but can also be exogenously given outside of the model such as political pressures or legislative restrictions.

the central bank would only intervene up to L_M ex-post, outsiders' concern becomes self-fulfilling. Hence, the lack of central bank's commitment can lead to unintended sub-optimal outcomes.

Bluffing under time inconsistency. We can also consider a case where the central bank faces a constraint on the amount of assets it can acquire. For instance, the central bank may not be an efficient user of these assets, where it can only generate $R - \Delta_{CB}$ from the assets. Denoting α_{CB} as the unit of assets the central bank acquires, suppose that Δ_{CB} is increasing in α_{CB}, that is, as the central bank acquires more assets, it starts to acquire assets it is less and less efficient in running. Also, running a large portfolio of assets can require additional resources from the central bank and can distract its main efforts in sustaining price and financial stability. Unlike quantitative easing, MMLR intends to buy assets temporarily and sell later when private markets recover, which would be harder to rewind with larger inventories. There may also be political pressures from purchasing too many assets since the central bank would be criticized for "replacing" the private market. For all these reasons, it may not be ex-post efficient or even implementable for the central bank to acquire more than $\hat{\alpha}_{CB}$ units of assets. Note that this would change the central bank loss function as follows:

$$\mathcal{L} = \gamma(\alpha) + E[\tilde{L}] + \delta(\alpha_{CB}) \times \mathbf{1}_{\alpha_{CB} > \hat{\alpha}_{CB}}, \tag{15}$$

where δ is positive and increasing, and $\mathbf{1}$ is the indicator function that equals 1 for $\alpha_{CB} > \hat{\alpha}_{CB}$, and 0 otherwise.

As with $\tilde{L}$, α_{CB} also depends on the amount of asset liquidations α at $t = 0$ and the insider liquidity I_1 at $t = 1$. In particular, we have:

- For $I_1 \geq \alpha R$, insiders have enough cash to pay R for all the assets at $t = 1$. Hence, all assets are acquired by the insiders and $\alpha_{CB} = 0$.
- For $\alpha R - L \leq I_1 < \alpha R$, the price is $P_1 = R$, where insiders acquire $\frac{I_1}{R}$ units and the rest is acquired by the central bank, that is, $\alpha_{CB} = \alpha - \frac{I_1}{R}$.
- For $\alpha(R - \Delta) - L \leq I_1 < \alpha R - L$, the price is $P_1 = \frac{L+I_1}{\alpha}$, which gives $\alpha_{CB} = \frac{L}{P_1} = \frac{\alpha L}{L+I_1}$.
- For $0 \leq I_1 < \alpha(R - \Delta) - L$, the price is $P_1 = R - \Delta$ even with the fully utilized central bank facility and we have $\alpha_{CB} = \frac{L}{R-\Delta}$.

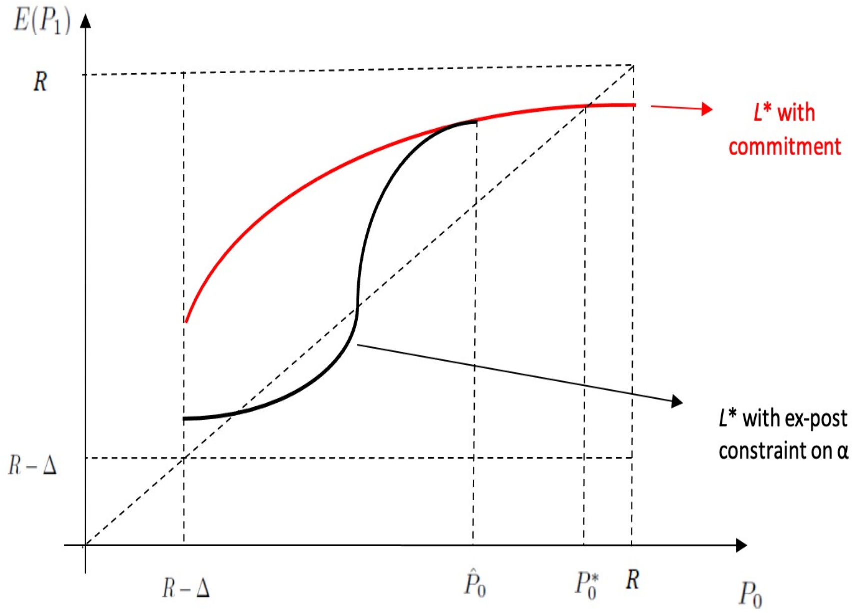

Figure 8: Figure illustrates the ex-post commitment problem with the interior solution L_1 as in Figure 6 and how the lack of commitment may impair its implementation.

This gives us:

$$\alpha_{CB} = \begin{cases} \frac{L}{R-\Delta} & \text{for} \quad 0 \leq I_1 < \alpha(R-\Delta) - L \\ \frac{\alpha L}{L+I_1} & \text{for} \quad \alpha(R-\Delta) - L < I_1 < \alpha R - L \\ \alpha - \frac{I_1}{R} & \text{for} \quad \alpha R - L \leq I_1 < \alpha R \\ 0 & \text{for} \quad I_1 > \alpha R \end{cases}. \tag{16}$$

Note that given L and I_1, α_{CB} is increasing in α — all else equal, the central bank would need to purchase more assets at $t = 1$ when more assets get liquidated at $t = 0$.

Figure 8 presents the commitment problem that would arise when the optimal policy is an interior solution L_1 as in Figure 6. Suppose the central bank has optimally chosen $L^* = L_1$ to minimize the loss $\mathcal{L}$. If the central bank can commit to implementing this, we would have the unique equilibrium P_0^* along with the corresponding α^*, and suppose that this is small enough to satisfy $\alpha^* < \hat{\alpha}_{CB}$. Here, the two loss functions given in (12) and (15) become equivalent with $\alpha^* < \hat{\alpha}_{CB}$ – the central bank might have bluffed but ex post it worked well due to the commitment since it never had to acquire more than $\hat{\alpha}_{CB}$.

Now suppose that the central bank cannot commit, and would need to restrict its asset acquisition ex post with the upper bound $\hat{\alpha}_{CB}$. This changes the shape of $E[P_1(P_0)]$ as in Figure 8. Since $\alpha(P_0)$ decreases continuously in P_0, there exists $\hat{P}_0$ such that $\alpha(P_0) = \hat{\alpha}_{CB}$ for $P_0 = \hat{P}_0$. The central bank would not need to acquire more than $\hat{\alpha}_{CB}$ units at $t = 1$ if $P_0 > \hat{P}_0$, but for $P_0 < \hat{P}_0$, it may be forced to limit its intervention below the announced capacity and thus, we see the deviation of the two curves below $\hat{P}_0$.

As Figure 8 illustrates, multiple fixed points can arise when the central bank has the ex-post constraint and cannot commit credibly ex-ante. As in the previous section, we have the "bad" equilibria in addition to the "good" equilibrium. In the good equilibrium, high P_0 leads to low α, which allows the central bank not to deviate from its announcement at $t = 1$. In the bad equilibrium, however, low P_0 leads to large liquidations α, which tests central bank's commitment and forces the central bank to deviate from its announcement at $t = 1$.

5 Discussion

In this section, we discuss the potential impact of MMLR operations on private incentives and alternative policy measures to restore liquidity.

5.1 Incentive distortions or Moral Hazard?

We next extend our baseline model to discuss whether the MMLR facility should remain in the central bank's permanent toolkit, like discount window lending, or make itself available only in special circumstances. We argue that the permanent availability of the facility may distort private incentives to hoard liquidity, thereby offsetting the benefit of the public backstop.[23]

In our baseline model, we consider the random arrival of insider liquidity at $t = 1$ (i.e., the distribution of I_1) as exogenously given. Suppose that this ex-post liquidity is influenced by private agents' ex-ante decisions. That is, liquidity providers choose how much liquidity to hoard at $t = -1$, and their decisions affect the $t = 1$ distribution of I_1 such that more private liquidity would be

[23] We assume a unique equilibrium in this subsection.

available ex-post if more is hoarded ex-ante.[24]

Next, we define the expected return for liquidity hoarding. Liquidity providers can use their cash I_1 at $t = 1$ to purchase assets from outsiders. Since they pay P_1 per unit generating R at $t = 2$, the expected return is given by

$$\bar{r} \equiv E\left(\frac{R}{P_1}\right) = \frac{R}{E[P_1]},$$

where the RHS decreases with liquidity hoarding since more liquidity would result in a higher expected price $E(P_1)$. Assuming risk-neutral liquidity hoarders with returns from outside options given by r_f, the excess return $\bar{r}$ must be equal to r_f in equilibrium to clear the market.

Suppose that the liquidity hoarders at $t = -1$ fully anticipate the MMLR intervention with capacity L to be announced at $t = 0$, where there arises a systemic liquidity event. It then has the following perverse effect.

Proposition 4. *If anticipated by liquidity hoarders, MMLR interventions reduce private liquidity hoarding, and we always have the same asset price P_0 regardless of the central bank's choice of L. The expected usage of the facility $E[\tilde{L}]$ and the central bank loss $\mathcal{L}$ increases in L.*

In other words, the central bank's willingness to provide liquidity constrains the profitability of private liquidity provision, impairing incentives to hoard liquidity ex-ante (e.g., Gale and Yorulmazer 2013; Choi et al. 2016). In equilibrium, public liquidity simply *replaces* private liquidity, such that the same amount of assets (i.e., same α) gets liquidated at $t = 0$. Without affecting the scale of fire-sales, the central banks need to spend more when announcing a larger facility capacity.[25]

This result underscores the potential downsides of incorporating the MMLR intervention into the permanent toolkit of central banks. Various measures should be taken to counter balance the incentive distortions it may create such maintaining some level of "constructive ambiguity" in its implementation. Additionally, MMLR can be accompanied by regulatory measures such as making

[24]For instance, more hoarding shifts the distribution of I_1 to the right.

[25]Note that we employed an extreme deterministic setup to illustrate the distortion most clearly. In a richer setup with uncertainty or a decreasing returns to scale technology for the return r_f, public liquidity may not fully replace private liquidity. Yet, even in those setups, private incentives to hoard liquidity would diminish, and the result would hold qualitatively.

the access to the facility conditional on certain solvency and liquidity criteria or require institutions to hold some level of liquidity ex ante.

Besides distorting the private liquidity provision incentives discussed above, containing disorderly liquidations may undermine the disciplining role of runs (Calomiris and Kahn, 1991; Diamond and Rajan, 2001). The affected institutions may rely excessively on the central bank and hold inadequate levels of liquidity (Repullo, 2005). In addition, by providing these institutions an option, MMLR may delay and prevent the cleaning up of their balance sheets (Diamond and Rajan, 2011; Acharya and Tuckman, 2014).

5.2 Comparison with other liquidity interventions

Next, we briefly discuss how the MMLR intervention in our setup differs from other central bank liquidity interventions.

Lender of the last resort (LoLR). Traditionally, central banks provide liquidity support to banks through their lending facilities. However, non-bank institutions like mutual funds or pension funds, considered insiders in our model, often face restrictions in accessing these facilities despite their growing presence and importance. Access for such "shadow banks" needs to be preceded by regulatory and supervisory reforms to constrain their moral hazard.

While central banks can still extend loans to banks through existing lending facilities, there is no guarantee that such liquidity will be swiftly transferred to non-banks (insiders in our model) in the presence of frictions in financial markets.[26] Furthermore, in our model, outsiders are hesitant to acquire assets from insiders not due to a shortage of funds, but rather because of concerns about being able to sell their inventory of assets in an market with limited liquidity at $t = 1$. In this context, simply channeling funds to outsiders at $t = 0$ or $t = 1$, via bank-dealers' access to lending facilities, does not influence their willingness to provide liquidity as market makers.

Asset purchases at $t = 0$. Instead of announcing to buy at $t = 1$, the central bank could opt to buy immediately at $t = 0$ alongside outsiders. While this measure would surely promote liquidity to constrain market disruptions, such a "first resort" intervention differs from the "last resort" approach.

[26]Note that we assumed frictions between insiders and outsiders, preventing outsiders from lending to insiders swiftly.

The MMLR is designed to provide a liquidity *backstop*, and ideally, the facility would not need to be utilized much. On the other hand, the immediate intervention entails actual purchases by the central bank to address liquidity shortages.

Fixing the price P_1 ex ante. To bolster future market liquidity, the central bank could opt for fixing the future price P_1 instead of announcing how much funds to use.[27] If successful, this policy should also affect P_0 to restore market liquidity, potentially obviating the need for future interventions by the central bank. However, adopting the price announcement may expose the central bank to a more significant commitment problem, as it imposes no limit on the extent of the actual intervention.

Furthermore, another concern arises from a potential lemons problem. By setting a fixed price, some institutions may exploit this situation to offload bad assets to the central bank, leading to an increased volume of sales at $t = 1$. Consequently, this would force the central bank to make additional purchases, further undermining its commitment to maintain the fixed purchase price.

6 Conclusion

After the crisis of 2007-2009, and of course, with the pandemic, the MMLR role of central banks attracted significant attention. Several central banks, including the Band of England, ECB, and the Federal Reserve, introduced MMLR facilities, and market liquidity promptly recovered even if actual asset purchases were minimal. However, similar interventions introduced by the Bank of England and ECB in late 2022 did not work as before.

This paper developed a theoretical framework to formalize the functioning of MMLR interventions on market liquidity, which allows us to examine its optimal design and robust implementation. Our results have the following policy implications. First, MMLR interventions must be forceful enough to obtain the intended outcome. Second, the central bank may get to intervene less ex-post when it announces a more aggressive intervention ex-ante. Third, interventions with insufficient capacity or lack of commitment by the central bank may result in multiple self-fulfilling equilibria

[27] Kuong (2020) and Li and Ma (2022) discuss how central banks providing a "price guarantee" can reduce risks associated with systemic runs.

to bring fragilities. Fourth, incorporating the MMLR into the permanent toolkit of central banks can pose potential downsides due to incentive distortion problems affecting private agents.

The model developed in the paper would enrich our understanding of this new policy option and provide a basis for future work that analyzes the success and fragility of such facilities, both theoretically and empirically. As commitment becomes critical, a pre-determined rule may help central banks resolve time-inconsistency problems. However, this may distort ex-ante incentives, where some sort of constructive ambiguity or accompanying regulatory measures would be necessary to prevent moral hazard. These are important issues that deserve further research.

References

Acharya, V. and B. Tuckman (2014). Unintended consequences of lolr facilities: The case of illiquid leverage. *IMF Economic Review 62*(4), 606–655.

Acharya, V. V., D. Gromb, and T. Yorulmazer (2012). Imperfect competition in the interbank market for liquidity as a rationale for central banking. *American Economic Journal: Macroeconomics 4*(2), 184–217.

Acharya, V. V., H. S. Shin, and T. Yorulmazer (2010). Crisis Resolution and Bank Liquidity. *Review of Financial Studies 24*(6), 2166–2205.

Acharya, V. V., H. S. Shin, and T. Yorulmazer (2013). A theory of arbitrage capital. *Review of Corporate Finance Studies 2*(1), 62–97.

Acharya, V. V. and T. Yorulmazer (2007). Cash-in-the-Market Pricing and Optimal Resolution of Bank Failures. *Review of Financial Studies 21*(6), 2705–2742.

Allen, F. and D. Gale (1994). Limited market participation and volatility of asset prices. *American Economic Review 84*(4), 933–955.

Allen, F. and D. Gale (1998). Optimal financial crises. *Journal of Finance 53*(4), 1245–1284.

Amihud, Y. and H. Mendelson (1980). Dealership market: Market-making with inventory. *Journal of Financial Economics 8*(1), 31–53.

Bagehot, W. (1873). *Lombard Street: A description of the money market.* HS King & Company.

Bao, J., M. O'Hara, and X. Zhou (2018). The volcker rule and corporate bond market making in times of stress. *Journal of Financial Economics 130*(1), 95–113.

Bessembinder, H., S. Jacobsen, W. Maxwell, and K. Venkataraman (2018). Capital commitment and illiquidity in corporate bonds. *Journal of Finance 73*(4), 1615–1661.

Bordo, M. D. (1990). The lender of last resort: Alternative views and historical experience. *FRB Richmond Economic Review 76*(1), 18–29.

Boyarchenko, N., A. Kovner, and O. Shachar (2021). It's what you say and what you buy: A holistic evaluation of the corporate credit facilities. Federal Reserve Bank of New York Staff Reports, no. 935.

Brunnermeier, M. and A. Krishnamurthy (2020). Corporate debt overhang and credit policy. *Brookings Papers on Economic Activity 2020*(2), 447–502.

Brunnermeier, M. K. and L. H. Pedersen (2009). Market liquidity and funding liquidity. *Review of Financial Studies 22*(6), 2201–2238.

Calomiris, C. W. and C. M. Kahn (1991). The role of demandable debt in structuring optimal banking arrangements. *American Economic Review 81*(3), 497–513.

Cecchetti, S. and P. Tucker (2021). Understanding how central banks use their balance sheets: A critical categorisation. VoxEU.org.

Chen, Q., I. Goldstein, and W. Jiang (2010). Payoff complementarities and financial fragility: Evidence from mutual fund outflows. *Journal of Financial Economics 97*(2), 239–262.

Choi, D. B., T. M. Eisenbach, and T. Yorulmazer (2016). Sooner or later: Timing of monetary policy with heterogeneous risk-taking. *American Economic Review 106*(5), 490–495.

Choi, J., Y. Huh, and S. Seunghun Shin (2023). Customer liquidity provision: Implications for corporate bond transaction costs. *Management Science forthcoming*, null.

Diamond, D. W. and R. G. Rajan (2001). Liquidity risk, liquidity creation, and financial fragility: A theory of banking. *Journal of Political Economy 109*(2), 287–327.

Diamond, D. W. and R. G. Rajan (2011). Fear of fire sales, illiquidity seeking, and credit freezes. *Quarterly Journal of Economics 126*(2), 557–591.

Duffie, D. (2010). Asset price dynamics with slow-moving capital. *Journal of Finance 65*(4), 1237–1267.

Ennis, H. M. (2016). Models of discount window lending: A review. *Economic Quarterly* (Issue 1Q), 1–50.

Falato, A., I. Goldstein, and A. Hortaçsu (2021). Financial fragility in the covid-19 crisis: The case of investment funds in corporate bond markets. *Journal of Monetary Economics 123*, 35–52.

Gale, D. and T. Yorulmazer (2013). Liquidity hoarding. *Theoretical Economics 8*(2), 291–324.

Goldberg, J. and Y. Nozawa (2021). Liquidity supply in the corporate bond market. *The Journal of Finance 76*(2), 755–796.

Goldstein, I., H. Jiang, and D. T. Ng (2017). Investor flows and fragility in corporate bond funds. *Journal of Financial Economics 126*(3), 592–613.

Gromb, D. and D. Vayanos (2002). Equilibrium and welfare in markets with financially constrained arbitrageurs. *Journal of Financial Economics 66*(2-3), 361–407.

Grossman, S. J. and M. H. Miller (1988). Liquidity and market structure. *Journal of Finance 43*(3), 617–633.

Haddad, V., A. Moreira, and T. Muir (2021). When selling becomes viral: Disruptions in debt markets in the covid-19 crisis and the fed's response. *Review of Financial Studies 34*(11), 5309–5351.

He, Z., S. Nagel, and Z. Song (2022). Treasury inconvenience yields during the covid-19 crisis. *Journal of Financial Economics 143*(1), 57–79.

Kargar, M., B. Lester, D. Lindsay, S. Liu, P.-O. Weill, and D. Zúñiga (2021). Corporate bond liquidity during the covid-19 crisis. *Review of Financial Studies 34*(11), 5352–5401.

Krugman, P. (2014). Even more on scotland. New York Times, 10 September 2014.

Kuong, J. C.-F. (2020). Self-Fulfilling Fire Sales: Fragility of Collateralized Short-Term Debt Markets. *Review of Financial Studies 34*(6), 2910–2948.

Li, Z. and K. Ma (2022). Contagious bank runs and committed liquidity support. *Management Science 68*(12), 9152–9174.

Ma, Y., K. Xiao, and Y. Zeng (2021). Mutual fund liquidity transformation and reverse flight to liquidity. Working Paper.

Mehrling, P. (2010). *The New Lombard Street: How the Fed became the Dealer of Last Resort.* Princeton University Press.

Mitchell, M., L. H. Pedersen, and T. Pulvino (2007). Slow moving capital. *American Economic Review 97*(2), 215–220.

O'Hara, M. and X. Zhou (2021). Anatomy of a liquidity crisis: Corporate bonds in the covid-19 crisis. *Journal of Financial Economics 142*(1), 46–68.

Repullo, R. (2005). Liquidity, risk taking, and the lender of last resort. *International Journal of Central Banking 1*(47-80).

Santos, J. A. (2006). Insuring banks against liquidity shocks: the role of deposit insurance and lending of last resort. *Journal of Economic Surveys 20*(3), 459–482.

Shleifer, A. and R. W. Vishny (1992). Liquidation values and debt capacity: A market equilibrium approach. *Journal of Finance 47*(4), 1343–1366.

Stein, J. C. (2012). Monetary policy as financial-stability regulation. *Quarterly Journal of Economics 127*(1), 57–95.

Stoll, H. R. (1978). The supply of dealer services in securities markets. *Journal of Finance 33*(4), 1133–1151.

Thornton, H. (1982). *An Enquiry into the Nature and Effects of the Paper Credit of Great Britain.* London. F.A. von Hayek (ed.). An Introduction. New York: Rinehart and Co., 1939.

Tucker, P. (2009). The repertoire of official sector interventions in the financial system: Last resort lending, market-making, and capital. In *Speech at the Bank of Japan 2009 International Conference.*

Vissing-Jorgensen, A. (2021). The treasury market in spring 2020 and the response of the federal reserve. *Journal of Monetary Economics 124*, 19–47.

Appendix

Proof of Proposition 2:

It is obvious that $-\Big[\frac{\partial E[P_1]}{\partial L} \times \frac{R\big[F\big(\min\{\alpha R, \overline{I}\}\big) - F(\alpha R - L)\big]}{F(\alpha R - L)} - \frac{\partial E[P_1]}{\partial \alpha}\Big]^{-1}$ is continuous in L because $\frac{\partial E[P_1]}{\partial L}$ and $\frac{\partial E[P_1]}{\partial \alpha}$ are continuous. We show that for the uniformly distributed I_1, this is increasing in L when $\alpha R - \overline{I} < L < \alpha R$. Note that here we have case (iii) for smaller L with $\alpha R - \overline{I} < L < \alpha(R - \Delta)$, and case (iv) for larger L with α(R-Δ)$<L < \alpha R$.

We first analyze case (iii). From (5) and (6), we have $\frac{\partial E[P_1]}{\partial L} = \frac{\Delta}{\overline{I}}$ and $\frac{\partial E[P_1]}{\partial \alpha} = -\frac{2\Delta R - \Delta^2}{2\overline{I}}$ in this case. The RHS of (10) hence becomes

$$RHS = -\Big[\frac{\Delta}{\overline{I}} \times R\frac{\min\{L, \overline{I} - (\alpha R - L)\}}{\alpha R - L} + \frac{2\Delta R - \Delta^2}{2\overline{I}}\}\Big]^{-1},$$

which is increasing in L.

We next analyze case (iv) where we have $\frac{\partial E[P_1]}{\partial L} = \frac{\alpha R - L}{\alpha \overline{I}}$ and $\frac{\partial E[P_1]}{\partial \alpha} = -\frac{(\alpha R + L)(\alpha R - L)}{2\alpha^2 \overline{I}}$. Hence, the RHS of (10) becomes

$$\begin{aligned} RHS = -\alpha\overline{I}\Big[R \times \min\{L, \overline{I} - (\alpha R - L)\} + \frac{(\alpha R + L)(\alpha R - L)}{2\alpha}\Big]^{-1} \\ = -\alpha\overline{I}\Big[-\frac{(\alpha R - L)^2}{2\alpha} + \alpha R^2 + R \times \min\{0, \overline{I} - \alpha R\}\Big]^{-1}, \end{aligned}$$

which is again increasing in L with $L < \alpha R$. The maximum and minimum come from the monotonically and continuity. ■

Proof of Proposition 3:

Note that we had different cases (i.e., case (i) to (v)) depending on the size of L. Since we would like to consider $E[P_1]$ as a function of P_0, we now need to solve for the boundaries for each case with respect to P_0. We can do this by first solving with respect to α, and then with respect to P_0 using the inverse function $P_0 = \alpha^{-1}$. We therefore have

$$\frac{\partial E[P_1]}{\partial P_0} = \begin{cases} 0 & \text{(i) if } P_0 < \alpha^{-1}\left(\frac{L+\bar{I}}{R-\Delta}\right) \\ -\int_{\alpha(R-\Delta)-L}^{\bar{I}} \frac{(L+I_1)\frac{d\alpha}{dP_0}}{\alpha^2} f(I_1)dI_1 & \text{(ii) if } \alpha^{-1}\left(\frac{L+\bar{I}}{R-\Delta}\right) < P_0 < \alpha^{-1}\left(\frac{L+\bar{I}}{R}\right) \\ -\int_{\alpha(R-\Delta)-L}^{\alpha R-L} \frac{(L+I_1)\frac{d\alpha}{dP_0}}{\alpha^2} f(I_1)dI_1 & \text{(iii) if } \alpha^{-1}\left(\frac{L+\bar{I}}{R}\right) < P_0 < \alpha^{-1}\left(\frac{L}{R-\Delta}\right) \\ -\int_{0}^{\alpha R-L} \frac{(L+I_1)\frac{d\alpha}{dP_0}}{\alpha^2} f(I_1)dI_1 & \text{(iv) if } \alpha^{-1}\left(\frac{L}{R-\Delta}\right) < P_0 < \alpha^{-1}\left(\frac{L}{R}\right) \\ 0 & \text{(v) if } P_0 > \alpha^{-1}\left(\frac{L}{R}\right) \end{cases},$$

and for the uniform I_1, this becomes

$$\frac{\partial E[P_1]}{\partial P_0} = \begin{cases} 0 & \text{(i) if } P_0 < \alpha^{-1}\left(\frac{L+\bar{I}}{R-\Delta}\right) \\ -\frac{d\alpha}{dP_0}\frac{1}{\alpha^2\bar{I}}\left[L\bar{I} + \frac{\bar{I}^2}{2} - \frac{(\alpha(R-\Delta)+L)(\alpha(R-\Delta)-L)}{2}\right] & \text{(ii) if } \alpha^{-1}\left(\frac{L+\bar{I}}{R-\Delta}\right) < P_0 < \alpha^{-1}\left(\frac{L+\bar{I}}{R}\right) \\ -\frac{d\alpha}{dP_0}\frac{\Delta}{\bar{I}}\left[R - \frac{\Delta}{2}\right] & \text{(iii) if } \alpha^{-1}\left(\frac{L+\bar{I}}{R}\right) < P_0 < \alpha^{-1}\left(\frac{L}{R-\Delta}\right) \\ -\frac{d\alpha}{dP_0}\frac{1}{2\bar{I}}\left[R^2 - \left(\frac{L}{\alpha}\right)^2\right] & \text{(iv) if } \alpha^{-1}\left(\frac{L}{R-\Delta}\right) < P_0 < \alpha^{-1}\left(\frac{L}{R}\right) \\ 0 & \text{(v) if } P_0 > \alpha^{-1}\left(\frac{L}{R}\right) \end{cases}.$$

Here it is clear that $\frac{\partial^2 E[P_1]}{\partial P_0^2} < 0$ for cases 3 and 4. Also, for linear α where $d\alpha/dP_0$ is constant, it's straightforward to show $\frac{\partial^2 E[P_1]}{\partial P_0^2} > 0$ for case 2. Considering the general case with $\frac{d^2\alpha}{dP_0^2} > 0$, for simplicity we assume $\alpha'''(P_0) \approx 0$, that is, $\frac{d^2\alpha}{dP_0^2}$ is constant.

We focus on case 2. Note that

$$\frac{\partial^2 E[P_1]}{\partial P_0^2} = \frac{\partial^2 E[P_1]}{\partial \alpha^2} \times \frac{d\alpha}{dP_0} + \left[\frac{\partial E[P_1]}{\partial \alpha} \times \frac{d^2\alpha}{dP_0^2}\right] \equiv A \times B + C \times D \tag{17}$$

where $A \equiv \frac{\partial^2 E[P_1]}{\partial \alpha^2} = \frac{d\alpha}{dP_0} \times 4\alpha\bar{I}[\bar{I}^2 + 2L\bar{I} - L^2] < 0$; $B \equiv \frac{d\alpha}{dP_0} < 0$; $C \equiv \frac{\partial E[P_1]}{\partial \alpha} \leq 0$; $D \equiv \frac{d^2\alpha}{dP_0^2} \equiv \kappa > 0$.

We now consider how these changes in P_0 for $\alpha^{-1}\left(\frac{L+\bar{I}}{R-\Delta}\right) < P_0 < \alpha^{-1}\left(\frac{L+\bar{I}}{R}\right)$. For the lowest P_0 with which $\alpha = \frac{L+\bar{I}}{R-\Delta}$, we have $P_0 = R - \Delta$ and thus $C = 0$. Hence, since $E[P_1]$ increases in P_1, we know $\frac{\partial^2 E[P_1]}{\partial P_0^2} > 0$ at its lower bound with the lowest $P_0 = R - \Delta$. Now, note that an increase in P_0 (and thus smaller α) would make (a) $|A|$ smaller, (b) $|B|$ smaller, (c) $|C|$ larger ($\because |\frac{\partial E[P_1]}{\partial \alpha}|$ decreases in α, see case 2 for $\frac{\partial E[P_1]}{\partial \alpha}$), (d) $|D|$ constant and unchanged. Hence, as P_0 increases, (17) decreases monotonically in this region. We can define $\hat{P}_0$ as the price that satisfies $A \times B + C \times D = 0$. If $A \times B + C \times D > 0$ for all $\alpha^{-1}\left(\frac{L+\bar{I}}{R-\Delta}\right) < P_0 < \alpha^{-1}\left(\frac{L+\bar{I}}{R}\right)$, then $\hat{P}_0$ is defined from $\alpha(\hat{P}_0) = \frac{L+\bar{I}}{R}$ (i.e., threshold between cases 2 and 3).

Now we show that $\hat{P}_0$ decreases in L. Note that B and D in (17) are independent of L. Also, note that $|A| = |\frac{d\alpha}{dP_0} \times 4\alpha\bar{I}[\bar{I}^2 + 2L\bar{I} - L^2]|$ decreases in L and $|C|$ increases in L. Therefore, for larger L, we need larger $|\frac{d\alpha}{dP_0}|$ at $P_0 = \hat{P}_0$ to have $A \times B + C \times D = 0$. Hence, $\hat{P}_0$ decreases in L. ■

Proof of Proposition 4:

Note that $\bar{r} = \frac{R}{E[P_1]}$ decreases in L, since $E[P_1]$ increases in L. Also note that $E[P_1]$ increases (decreases) when the distribution of I_1 shifts to the right (left).

In equilibrium, the indifference condition $\bar{r} = r_f$ needs to hold for all L to clear the market. Therefore, an increase (decrease) in L induces less insider liquidity hoarding, i.e., the distribution of I_1 to shift to the left (right), so as to satisfy the market clearing condition $E[P_1] = \frac{R}{r_f}$ for any L.

Since $P_0 = E[P_1]$, this implies P_0 has the same value for any L with private liquidity hoarding getting adjusted, which in turn indicates the same $\alpha = \alpha(P_0)$ for all L. A shift of the distribution of I_1 to the left induced by a larger L, combined with the same α, indicates a larger $E[\tilde{L}]$, and thus a larger $\mathcal{L}$. ■