The Central Influencer Theorem: Spatial Voting Contests with Endogenous Coalition Formation *

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Abstract

We analyze a spatial voting contest without the "one person, one vote" restriction. Players exert continuous influence effort and incurs cost accordingly. They can be heterogeneous in terms of position, disutility function, and cost function. In equilibrium, two groups endogenously emerge: players in one group try to implement more leftist policy, while those in the other group more rightist one. Since the larger group suffers more severe free-riding problem, the equilibrium policy does not converge to the center if the larger group does not have a cost advantage. We demonstrate how the location of the center (i.e., the steady-state point) depends the convexities of the utility and cost functions. We extend the model to a dynamic setting.

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1. Introduction

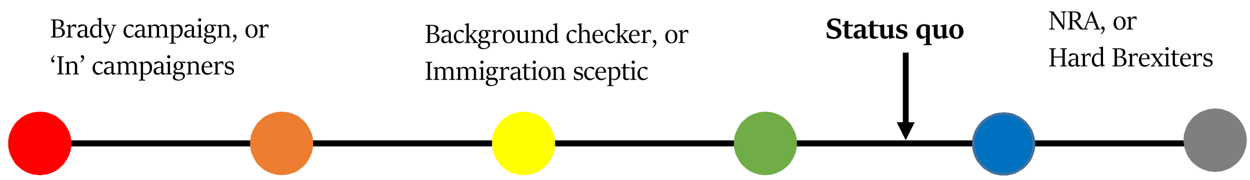
In many a times a policy in the government, organizations, committees etc. is chosen and implemented through (majoritarian) voting. In general, there exists a status quo policy and the players involved have their own preferences regarding what a new policy should be while voting on the policy change. This generic structure can be implemented in other collective decision making processes such as election, lobbying etc. and has been applied, bot theoretically and empirically, numerous times in Economics and Political Science.

When the policy is considered in a single dimension, this structure provides the famous 'median voter' result (Black, 1948; Downs, 1957). In specific, the players are located in the linear policy space according to their preferred policy point. The players suffer with disutility if the policy is far from their preferred point and cast their single possible vote. Under this structure, it is shown in various studies that the preference of the median voter is reflected in the final policy.

Note, however, that very many real life situations go beyond such simple structure even while considering a single dimensional policy space. First, often a committee or the members of public essentially engage in contests by exerting costly efforts to influence policies. Here the one person one vote rule is not applicable. For example, in the case of rent seeking, electoral expenditures, lobbying in a committee etc., the influence procedure is rather general and involve associated costs. Moreover, to implement one's favorable policy or to influence the status quo, players often form coalitions endogenously with other 'like-minded' players. Finally, some members of the coalition may free-ride on other members while influencing for such favorable policies.

In this study we introduce a game theoretic model to combine the above discussed features in both static and dynamic settings and derive the resulting equilibria. The model turns out, as we show later, to be a combination of a spatial voting model and a collective contest model. In this setting the players have their optimal preference over a line with a given disutility function: further the implemented policy, higher is the dissatisfaction. The initial status quo policy is given exogenously (from history), but the players can spend efforts to influence the policy. The efforts are costly and follows a known cost function. If the players from the left (right) side of the initial policy collectively exert more effort, then the final policy is more likely to be implemented in the left (right) side of the initial policy. Hence, in doing so, players form coalitions endogenously and are able to free-ride on other players in such a coalition. Below, in Figure 1, we provide an illustrative example of this situation.

Figure 1. Illustrative example



Suppose the line above illustrates the policy space. We provide two examples: first, the policy can be the policy on gun-control in the USA. A player with the extreme preference in the right (grey circle) here would be the NRA, who would like to promote and encourage rifle shooting. Another player with the extreme preference in the left (red circle) here would be the Brady campaign who would prefer for gun control and stop gun violence. Somewhere in the middle (say, the yellow circle) would be someone who is pro-gun ownership, but prefer stricter background checks. Likewise, for the case of Brexit in the UK, the grey, red, and yellow circles will depict the 'hard Brexiters', the 'In campaigners, and the soft immigrations sceptics. Of course, there are many other players (blue, green, orange etc.) who hold their own preferences in both the examples.

Suppose further, that the black arrow specifies the status quo. Then, the yellow player would like the new policy to be moved towards left. Note a few interesting aspects that differentiates this structure with the standard voting models. First, although yellow wants the policy to move towards left, s/he does not want it to move too left. Second, implicitly yellow makes an alliance with the green, orange, and red players, since those players also want to move he policy towards left. Third, if the green, orange, and red players exert costly effort to move the policy, then it is possible for yellow to free-ride on their effort and refrain from exerting costly effort.

The players can be heterogeneous in terms of three aspects: their position, their effort cost function, and their disutility function. We use a very specific logit-type (Tullock, 1980) adjustment function for the policy change. We find that in equilibrium, two groups endogenously emerge: players in one group try to implement more leftist policy, while those in the other group more rightist one. In general, the equilibrium policy converges to the 'center' if the larger of the groups has a cost advantage. However, it may not converge if there is no such an advantage because the larger group suffers with a more severe free-rider problem. The 'center', however, does not have to be the median; but it can be other measures of centrality.

We define steady state equilibrium as the situation in which the status quo and the implemented policy coincide. We find that if both the disutility of non-optimal policy and the effort cost function are linear, any policy can be a steady state policy. This is the case where there is no cost (dis)advantage. However, if the disutility of non-optimal policy is linear and the cost function is convex, a steady state policy is the optimal policy for a median player. On the contrary, if the disutility of non-optimal policy is concave (that is, the distance measure is convex) and the cost function is linear, the steady state policy is the mean of the two extreme players' optimal policies. There is no cost advantage in this case, but everyone but the two extreme players free-ride in equilibrium. Finally, if the distance measure and the cost function are equally convex, the steady-state policy is the (weighted) average of all players' ideal points.

We then extend these analyses to a three player dynamic setting. In an infinite horizon model, there exists an equilibrium in which the policy converges again to the median player. Players expend more effort in each period as they become more patient (or forward-looking), but the convergence speed does not depend on the discount factor.

Our results contribute to the spatial voting literature and show that although a central measure turns out to be the optimal policy, it does not necessarily have to be the median. Krasa and Polborn (2010) in a very different setting consider competition between heterogeneous candidates on two policy areas. They also find a central tendency that is different from standard median voter. We contribute further in the same line. It also introduces the idea of contest in the spatial voting literature. Downs (1957) and the following studies (e.g. Palfrey and Rosenthal, 1983; Becker, 1983; Sengupta and Sengupta, 2008 etc.) assume that each player has the same influence (one vote). In the current study we allow each individual to spend more or less resources, thus the influence of an individual is endogenously determined.[1] This also supports and complements the results in Hanson and Stuart (1984) found in a different set up. In a generalization, Baron (1996) implements a collective goods problem through dynamic voting. Our approach is very close to this analysis, but we show a broader set of results.

We also contribute to the contest literature (Konrad, 2009) – especially in the areas of collective contests and endogenous coalition formation. In the collective (group) contests, players are

[1] Lalley and Weyl (2018) propose quadratic voting as a method for binary collective decision making, our model does not assume that two alternatives are exogenously given.

assumed to be in pre-specified groups in which the prize value is given, and there is exogenously given group production function (impact function). In our structure, we consider an additive impact function similar to that of Katz et al. (1990) but endogenize both the group size and the prize value. Individual contest literature does not consider a specific dimension in policy implementation, whereas coalition formation rules are often arbitrarily imposed (see, for example the references in Balart et al., 2017). We consider the linear policy dimension and contribute to the thin literature on contests with networks (Franke and Öztürk, 2015).[2] For examples, the Tug-of-war games are two-player games in which the policy moves sequentially until it reaches a pre-specified point (Konrad and Kovenock, 2005, 2009; Agastya and McAfee, 2006). Our model is a differentiated version in the sense that we consider multiple players and an endogenous final point. Duggan and Gao (2020) analyse a multi-dimensional tug-of-war model in which risk averseness result in Rawlsian equilibrium and risk loving equilibrium is the mean of players' ideal points. Compared to them we implement single dimension policy and risk neutrality, but consider groups.

Epstein and Nitzan (2004) introduce a two stage game. In the first stage the "groups decide which policy to lobby for and then, in a second stage, engage in a contest over the proposed policies". Hence, they endogenize the 'target' but not the prize. We extend that part of the study by also endogenizing the groups, but the preference points in our model remains fixed. Similarly, Baik (2017) considers a multi-players contest on a prize that is public good for some and public bad for some others, but only the prize spread matters in equilibrium that involves free riding. In contrast to this model, we endogenize the prize as well as the group formation.

In summary, we believe that the current study ties the spatial voting model and the collective contests model and contributes to both parts of the literature. In the continuation we first introduce the model in Section 2. Section 3 and Section 4 reports the results from the static and the dynamic analyses, respectively. Section 5 concludes.

2. Model

Consider $N \geq 2$ players who spend effort to implement an individually more desirable policy. The policy space is bounded, continuous and one-dimensional. Such a space can be represented as a unit interval $[0,1]$. Let x_i be the effort exerted by player i, $d_i \in \{-1,1\}$ be the direction toward which the effort is put, and $y_i \in [0,1]$ be the ideal point of player i. For expositional

[2] This is different from Spatial contests (Konrad, 2000), in which firms contest with each other for locations in a differentiated market.

simplicity, we assume that all players are distinct in terms of their ideal points, but the distance between them can be arbitrary.[3] More specifically and without loss of generality, we let $0 = y_1 < y_2 < \cdots < y_N = 1$. Player i decides on the pair (x_i, d_i) to maximize:

$$u_i(\delta, x_i) = -\alpha_i \|\delta - y_i\| - c_i(x_i) \tag{1}$$

where α_i is a parameter capturing the sensitivity of the player to a change in policy, and the cost of effort $c_i(x_i)$ is $\mu_i(x^\gamma/\gamma)$ where $\gamma \geq 1$. We further assume the distance measure $\|.\|$ to have the same functional form as the cost function, i.e., $\|\delta - y_i\| = (|\delta - y_i|)^\lambda/\lambda$ where $\lambda \geq 1$.[4] Also, we mostly restrict our focus on symmetric players, i.e., for all i and j, $\alpha_i = \alpha_j = \alpha$ and $\mu_i = \mu_j = 1$. The consequences of relaxing this symmetry assumption are discussed at the end of Subsection 4.2.

The implemented policy is determined according to an 'adjustment rule', which is defined as follows. Let $\boldsymbol{x}$ and $\boldsymbol{d}$ denote the vectors of effort and directions, respectively. Then, given $(\boldsymbol{x}, \boldsymbol{d})$, the implemented policy is:

$$\delta(\boldsymbol{x}, \boldsymbol{d}) = S + p(\boldsymbol{x}, \boldsymbol{d})$$

where S is the status quo or the default policy, and $p(\boldsymbol{x}, \boldsymbol{d})$ is the adjustment. In words, the newly implemented policy is the status quo policy adjusted by the aggregated efforts. Our model allows S to be different from previously implemented policy. But when discussing the dynamics and steady state equilibrium, we assume that the status quo at period t is the implemented policy at period $t-1$.

The adjustment function $p(\boldsymbol{x}, \boldsymbol{d}) \in [-1/2, 1/2]$ has the following form:

$$p(\boldsymbol{x}, \boldsymbol{d}) = p\left(\sum_{j \in \hat{L}} x_j, \sum_{j \in \hat{R}} x_j\right)$$

where $\hat{L}$ denotes the set of players who push the policy to the left, i.e., $\hat{L} = \{j | d_j = -1\}$ and similarly, $\hat{R} = \{j | d_j = 1\}$, and has the following properties:

i. $sgn[\partial p(\boldsymbol{x}, \boldsymbol{d})/\partial x_i] = sgn[d_i]$ and $sgn[\partial^2 p(\boldsymbol{x}, \boldsymbol{d})/\partial x_i^2] = -sgn[d_i]$.
ii. If $\sum_{j=1}^{N} d_j x_j = 0$, then $p(\boldsymbol{x}, \boldsymbol{d}) = 0$.

[3] If two or more players share the same ideal point (i.e., $y_i = y_j$ for some $i \neq j$), there may exist multiple equilibria in which those players free-ride on each other's effort in various ways. We ignore these cases because they would make the exposition significantly messier without adding any interesting insights.
[4] Provide reference for such function used in the literature and why that is fine to have the same function for both.

iii. If $\sum_{j=1}^{N} d_j x_j = 0$, then $\frac{\partial p(\boldsymbol{x},\boldsymbol{d})}{\partial x_i} = -\partial p(\boldsymbol{x},\boldsymbol{d})/\partial x_j$ for $i \in \hat{L}$ and $j \in \hat{R}$.

The first is the usual assumption that the function is increasing and concave in effort. It looks different from the usual one because the objective of the players in $\hat{L}$ is to reduce $p(\boldsymbol{x},\boldsymbol{d})$. The second assumption states that if the amount of efforts put forward the opposite directions are identical (i.e., $\sum_{j\in\hat{L}} x_j = \sum_{j\in\hat{R}} x_j$)), then the default policy is implemented. It also implies that if nobody exerts a positive effort, then $\delta(\boldsymbol{x},\boldsymbol{d}) = \mathrm{S}$. The last assumption is that $p(\boldsymbol{x},\boldsymbol{d})$ is symmetric.[5] More specifically, at a symmetric point (i.e., when $\sum_{j\in\hat{L}} x_j = \sum_{j\in\hat{R}} x_j$), the marginal change of the policy is also symmetric. Our leading example for $p(\boldsymbol{x},\boldsymbol{d})$ is a 'Contest Success Function (CSF)' in the spirit of Tullock (1980):

$$p(\boldsymbol{x},\boldsymbol{d}) = \begin{cases} \frac{\sum_{j=1}^{N} d_j x_j}{2\sum_{j=1}^{N} x_j} & if\ \sum_{j=1}^{N} x_j > 0 \\ 0 & \text{Otherwise} \end{cases} \quad (2).$$

From this, it is clear that out model is a collective rent-seeking game (similar to Katz et al., 1990), which is played on a single-dimensional policy space. Another example is a linear function: $p(\boldsymbol{x},\boldsymbol{d}) = \eta\left(\sum_{j=1}^{N} d_j x_j\right)$ for some positive but small η.

The ideal points of all players $\{y_i\}_{i=1}^{N}$ and the status quo policy S are common knowledge. All players decide the effort level and the direction $\{x_i, d_i\}_{i=1}^{N}$ independently and simultaneously. An equilibrium is vectors of efforts and directions $(\boldsymbol{x}^*, \boldsymbol{d}^*)$ such that for all i, given $(\boldsymbol{x}_{-i}^*, \boldsymbol{d}_{-i}^*)$ and S, player i maximizes (1).

3. Static Analysis

We first characterize the condition under which given $\boldsymbol{x}$, nobody has an incentive to change the direction of effort, then we explore how the equilibrium efforts determine the implemented policy. In Section 3.2, steady-state equilibria in which the status quo and the implemented policy coincide are characterized. Note that, if $\gamma = \lambda = 1$, i.e., both the distance and the effort cost functions are linear, then any policy $\delta \in [0,1]$ can be a steady-state equilibrium. In contrast, if the cost function is convex while the distance function is linear, i.e., $\gamma > 1$ and $\lambda = 1$, a steady state equilibrium policy must be in $[y_{m-}, y_{m+}]$ where y_{m-} is the ideal point of the

[5] Note that the last assumption does not directly imply that the marginal benefit t of exerting additional effort would be the same for those players, because in principle, both α_i and $|\delta - y_i|$ can influence the marginal utility of having δ closer to y_i.

left median player and y_{m+} is the idea point of the right median player. Of course, when N is an odd number then $y_{m-} = y_{m+} = y_m$. If $\gamma = 1$ and $\lambda > 1$, the mean of the two extreme players' ideal points, ½. Emerges as the steady state point. Finally, if $\gamma > 1$ and $\lambda > 1$, then mean of all players' ideal points is the steady state.

3.1 Group formation

In this subsection, we consider how groups are formed, that is, given the vector of efforts $\boldsymbol{x}$, how that of the directions $\boldsymbol{d}$ is determined. The following lemma describes what the groups $\hat{L} = \{j|d_j = -1\}$ and $\hat{R} = \{j|d_j = 1\}$ look like in equilibrium.

Lemma 1. *In equilibrium, there exists a threshold (or grouping rule)* $\theta \in [0,1]$ *such that the players whose ideal policy is in the left of* θ *are in group* $\hat{L}$*, and those who are in the right are in group* $\hat{R}$*. The player whose ideal policy is* θ*, if exists, can be in either group.*

Proof: Obvious.

We define $L(\theta)$ as the set of players who are at the left side of θ, i.e., $L(\theta) = \{i|y_i < \theta\}$, and $R(\theta)$ as $R(\theta) = \{i|y_i \geq \theta\}$. Since now we can infer the vector of directions $\boldsymbol{d}$ from $(\boldsymbol{y}, \theta)$, below we discuss how to determine an equilibrium threshold θ^* instead of the vector of directions $\boldsymbol{d}^*$.

For the sake of concreteness of the discussion, let $p(\boldsymbol{x}, \boldsymbol{d})$ be the Tullock-type CSF defined in (2) for a moment.[6] Note that since $y_1 = 0$ and $y_N = 1$, $\sum_j x_j$ is never zero in equilibrium. Therefore, the implemented policy as a function of θ is given by:

$$\delta(\theta; \boldsymbol{x}, \boldsymbol{y}, S) = S + \frac{\sum_{j \in R(\theta)} x_j - \sum_{j \in L(\theta)} x_j}{2 \sum_{j \in L(\theta) \cup R(\theta)} x_j}$$

$$= S - \frac{1}{2} + \frac{\sum_{j \in R(\theta)} x_j}{2 \sum_{j \in L(\theta) \cup R(\theta)} x_j} \qquad (3).$$

Note that given $\boldsymbol{x}, \boldsymbol{y}$ and S, the implemented policy $\delta(\theta)$ is a decreasing step function: as θ moves from 0 to 1, more and more players move from $R(\theta)$ to $L(\theta)$, so $\delta(\theta)$ decreases step by step.

[6] Except for the ones presented in Section 4.3, our results do not require any specific functional form assumption on $p(\boldsymbol{x}, \boldsymbol{d})$.

If the threshold θ is too small, too many players are on the right side of it, so the implemented policy ends up being biased toward the right. In such a case, a player in $R(\theta)$ but located close to θ has an incentive to change the direction of the effort from the right ($d = 1$) to the left ($d = -1$). If too many players are in $L(\theta)$, similarly, the implemented policy is biased towards the left, and a player located close to θ is willing to change the sides, i.e., θ must be moving towards the left. In equilibrium, the threshold must be set in a way such that nobody gains by changing her direction. The following lemma states that an equilibrium policy $\delta(\boldsymbol{x}^*, \boldsymbol{d}^*; S)$ is such a threshold.

Lemma 2. *Given a vector of equilibrium effort $\boldsymbol{x}^*$, the corresponding equilibrium groups are $L(\theta^*)$ and $R(\theta^*)$ where θ^* satisfies:*

$$\theta^* = \delta(\theta^*; \boldsymbol{x}^*, \boldsymbol{y}, S). \tag{4}$$

Proof: Consider an arbitrary grouping rule $\theta_0 \in (0,1)$ according to which players on the left of θ_0 are in L, and those on the right of or on θ_0 are in R. Suppose that $\delta(\theta_0; \boldsymbol{x}^*, \boldsymbol{y}) > \theta_0$, and that when the threshold moves from $\theta_0 < y_i$ to $\theta_1 > y_i$, $\delta(\theta_1; \boldsymbol{x}^*, \boldsymbol{y})$ is still greater than θ_1. Then, the change from θ_0 to θ_1 (equivalently, from $d_i = 1$ to $d_i = -1$) improves the utility of player i because by the change, $||\delta - y_i||$ becomes smaller. This means that θ_0 is not an equilibrium threshold, and furthermore, any $\theta < \theta_1$ is not an equilibrium threshold either. We can say the same thing about the case in which θ_0 is greater than δ, and moving θ_0 toward the left does not change the rank of the two. Because in equilibrium, nobody has an incentive to change the direction of the effort, the equilibrium dividing rule $\hat{\theta}$ must satisfy

$$\lim_{\theta \to \hat{\theta}_+} \delta(\theta; \boldsymbol{x}^*, \boldsymbol{y}, S) \leq \hat{\theta} \leq \lim_{\theta \to \hat{\theta}_-} \delta(\theta; \boldsymbol{x}^*, \boldsymbol{y}, S),$$

and the equilibrium implemented policy is either the right limit or the left limit.

To prove the lemma by contradiction, suppose that the above inequalities are strict, which implies that there exists a point $y_j = \hat{\theta}$ such that when threshold θ is on the left of y_j, $\theta < \delta(\theta; \boldsymbol{x}^*, \boldsymbol{y}, S)$, but $\theta > \delta(\theta; \boldsymbol{x}^*, \boldsymbol{y}, S)$ when θ is on the right of y_j. Graphically, y_j is the threshold in which $\delta(\theta)$ jumps from the above of 45 degree line to the below of it. In this case, player j can pull the implemented policy $\delta(x_j, \boldsymbol{x}_{-j}^*; \boldsymbol{y})$ toward her ideal policy y_j by reducing her effort x_j. This means $\boldsymbol{x}^*$ was not a vector of equilibrium efforts in the first place, which contradicts the assumption. Therefore, in equilibrium at least one of the inequalities must hold as equality, and the equilibrium policy must be the dividing rule which satisfies $\theta^* = \delta(\theta^*; \boldsymbol{x}^*, \boldsymbol{y}, S)$. ■

A few remarks follow immediately. First, the equilibrium grouping rule θ^* defined by Coate (2004) is unique if exists, because $\delta(\theta)$ is a decreasing step function. This, of course, does not mean that the equilibrium is unique. Second, even if the equilibrium groups are unique, there can be infinitely many thresholds θ that define the same groups. Third, it is δ^* not S that determines the directions of efforts: even when S is on the left of y_i, player may prefer push the policy to the left if the equilibrium policy δ^* ends up being on the right of y_i. Lastly, because $\delta(\theta; S)$ increases as S gets larger, given (x, θ^*) is non-decreasing in S.

3.2. Steady State

In this subsection, we characterize equilibria where $\delta^* = S$, namely steady-state equilibria. In such an equilibrium, the equilibrium groups are simply defined as $L(S)$ and $R(S)$. Given that the directions are set in the way to maximize each individual's utility, the game boils down to a simple collective-rent seeking game or a group contest.

Again, for concreteness, let us consider the Tullock (1980) CSF and assume $\lambda = 1$. Taking the constants out of Eq. (1), the maximization problem of player i in $Q(S)$ $(where\ Q = L, R)$ can be rewritten as:

$$\max_{x_i} \frac{\alpha \sum_{j \in Q} x_j}{\sum_{j \in L \cup R} x_j} - c(x_i).$$

Note that this objective function is identical to that in group contests with the value of the prize being α (see Katz et al., 1990). Since a player can "win a (public-good) prize" even if she exerts zero effort, players have an incentive to free-ride on the efforts of the other players in the same group. The first-order condition for player i in $Q(S)$ is:

$$\frac{\alpha \sum_{j \in Q^c} x_j^*}{\left(\sum_{j \in L \cup R} x_j^*\right)^2} - c'(x_i^*) \geq 0$$

where the inequality condition is for a player who would choose $x_i = 0$ because she is completely satisfied with the steady-state policy (i.e., $y_i = S = \delta^*$). In other words, in equilibrium, the FOCs hold as equality whenever $x_i^* > 0$. Recall that the implemented policy coincides with the status quo if and only if $\sum_{j \in L(S)} x_j^* = \sum_{j \in R(S)} x_j^*$. Using this, we derive the following conditions: for all $x_i^* > 0$,

$$\frac{\alpha}{4\sum_{j\in Q} x_j^*} = c'(x_i^*) = (x_i^*)^{\gamma-1}. \tag{5}$$

Equation (5) shows that the determination of the steady-state efforts and the corresponding policy crucially depend on the (non-)linearity of the cost function. If it is linear ($\gamma = 1$), all the FOCs are identical to each other, so there is a large indeterminacy. In contrast, if it is convex ($\gamma > 1$), regardless of how convex it is, everybody has to expend the same amount of effort in a steady-state equilibrium.

Now, consider the case with a convex distance measure, i.e., $\lambda > 1$. Given that $\delta^* = S$, the FOC of player i's maximization problem is

$$\alpha(|S - y_i|)^{\lambda-1} \frac{\alpha \sum_{j\in Q^c} x_j^*}{\left(\sum_{j\in L\cup R} x_j^*\right)^2} - (x_i^*)^{\lambda-1} \geq 0.$$

Let us first consider the case of the linear cost function ($\gamma = 1$). Since in a steady-state, $\alpha \sum_{j\in Q^c} x_j^* / \left(\sum_{j\in L\cup R} x_j^*\right)^2$ is common to every player, the players with the largest $\alpha(|S - y_i|)^{\lambda-1}$, that is, those farthest from S expends a positive effort, while the others free-ride. Because S must be in between 0 and 1, the players farthest from S in each group are those at the extremes, players 1 and N. In order for the FOCs of the extreme players to simultaneously hold as equality, $|S - y_1|$ must equal $|S - y_N|$. Therefore, $S = (y_1 + y_N)/2 = 1/2$.

Next, suppose that the cost function is also strictly convex ($\gamma > 1$). Notice that equilibrium effort x_i^* is $(|S - y_i|)^{\frac{\lambda-1}{\gamma-1}}$ multiplied by $\left[\frac{\alpha \sum_{j\in Q^c} x_j^*}{\left(\sum_{j\in L\cup R} x_j^*\right)^2}\right]^{1/(\gamma-1)}$ which is a factor common to everybody. Thus, for $\sum_{j\in L} x_j^* = \sum_{j\in R} x_j^*$ to be the case, $\sum_{j\in L}\left(|S - y_j|\right)^{\frac{\lambda-1}{\gamma-1}}$ must equal $\sum_{j\in R}\left(|S - y_j|\right)^{\frac{\lambda-1}{\gamma-1}}$. Suppose the distance measure and the cost function are convex by the same degree, i.e., $\gamma = \lambda$. Then, by equating $\sum_{j\in L}\left(S - y_j\right)$ and $\sum_{j\in R}\left(S - y_j\right)$, we conclude that in such a case, $S = \sum_{j=1}^{N} y_j / N$. The above logic is valid for a more general adjustment function $p(\boldsymbol{x}, \boldsymbol{d})$, and the discussion thus far is summarized in the following proposition.

Proposition 1. Suppose that a steady-state equilibrium exists.

(i) If $\gamma = \lambda = 1$, any point in $[0,1]$ can be a steady-state equilibrium policy.

(ii) If $\gamma > 1$ and $\lambda = 1$, a steady-state policy is the median player's ideal policy. That is, if N is an odd number, the steady-state policy must be y_m, and for N an even number, any point in $[y_{m-}, y_{m+}]$ can be a steady-state policy.

(iii) If $\gamma = 1$ and $\lambda > 1$, the steady-state policy is $1/2$.

(iv) If $\gamma = \lambda > 0$, the steady-state policy is the average of all ideal policies, $\sum_{j=1}^{N} y_j/N$.

Proof. Note first that for a more general $p(\boldsymbol{x}, \boldsymbol{d})$, which can also be written as $p(\boldsymbol{x}, \theta)$, for a proper θ by Lemma 1, the FOC is

$$\alpha(|S - y_i|)^{\lambda-1} \left| \frac{\partial p(x_i, \boldsymbol{x}_{-i}^*, \theta^*)}{\partial x_i} |_{x_i = x_i^*} \right| - (x_i^*)^{\lambda-1} \geq 0.$$

(i) Suppose $\gamma = \lambda = 1$. Since a steady-state equilibrium exists by assumption, there exists a tuple $(\boldsymbol{x}^*, \theta^*)$ that satisfies the FOCs. Now, pick an arbitrary status quo $S \in [0,1]$, let the groups be $L(S)$ and $R(S)$. Pick a vector $\boldsymbol{x}^{**}$ such that

$$\sum\nolimits_{j \in R(S)} x_j^{**} = \sum\nolimits_{j \in R(\theta^*)} x_j^*$$

and

$$\sum\nolimits_{j \in L(S)} x_j^{**} = \sum\nolimits_{j \in L(\theta^*)} x_j^*.$$

Then, since $p(\boldsymbol{x}, \theta) = p\left(\sum_{j \in L(\theta)} x_j, \sum_{j \in R(\theta)} x_j\right)$, in other words, since the adjustment function depends only on the sums of efforts, $\left|\frac{\partial p(x_i, \boldsymbol{x}_{-i}^*, \theta^*)}{\partial x_i} |_{x_i = x_i^*}\right| = \left|\frac{\partial p(x_i, \boldsymbol{x}_{-i}^{**}, S)}{\partial x_i} |_{x_i = x_i^{**}}\right|$. Since the marginal cost of exerting effort is independent of the size (i.e., the RHS of the FOC is constant), the vector of efforts x^{**} also satisfies the FOCs, and $\sum_{j \in R(S)} x_j^{**} = \sum_{j \in L(S)} x_j^{**}$ by definition. Hence, x^{**} together with the status quo S also constitutes a steady-state equilibrium. ∎

(ii) Now suppose $\gamma > 1$ and $\lambda = 1$. Because at a symmetric point (when $\sum_{j \in R} x_j = \sum_{j \in L} x_j$) the marginal change $\frac{\partial p(x, \theta)}{\partial x_i}$ is identical to everybody, the first-order condition implies that the equilibrium effort, too, must be identical, unless $x_i^* = 0$. Note that since $\lim_{x \to 0} c'(x) = 0$, $\lim_{x \to 0} u_i'(x, \delta) > 0$, which implies everybody but the player with $y_i = S = \delta^*$ prefers to exert a positive amount of effort. Since the equilibrium effort level is identical across the players, in order for the sums of efforts to be equal to each other, there should be an equal number of players in each group. This implies that as long as S divides the players into two symmetric groups, S can be a steady-state policy. ∎

(iii) Next, consider the case where $\gamma = 1$ and $\lambda > 1$. Since in a steady-state, $\frac{\partial p(x_i, \boldsymbol{x}^*_{-i}, \theta^*)}{\partial x_i}|_{x_i = x_i^*}$ is common to every player, the players with the largest $\alpha(|S - y_i|)^{\lambda-1}$, that is, those farthest from S expends a positive effort, while the others free-ride. Because S must be in between 0 and 1, the players farthest from S in each group are those at the extremes, players 1 and N. In order for the FOCs of the extreme players to simultaneously hold as equality, $|S - y_1|$ must equal $|S - y_N|$. Therefore, $S = (y_1 + y_N)/2 = 1/2$. ∎

(iv) Lastly, suppose that $\gamma = \lambda > 0$. From the FOC, we derive the following: if $x_i^* > 0$,

$$x_i^* = |S - y_N|.\left[\alpha\left|\frac{\partial p(x_i, \boldsymbol{x}^*_{-i}, \theta^*)}{\partial x_i}|_{x_i = x_i^*}\right|\right]^{1/(\gamma-1)}$$

Because $\frac{\partial p(x_i, \boldsymbol{x}^*_{-i}, \theta^*)}{\partial x_i}|_{x_i = x_i^*}$ is common to every player, for $\sum_{j\in R} x_j^* = \sum_{j\in L} x_j^*$ to be the case, $\sum_{j\in L}|S - y_j|$ must equal $\sum_{j\in R}|S - y_j|$. By equating $\sum_{j\in L}(S - y_j)$ and $\sum_{j\in R}(y_j - S)$, we conclude that $S = \sum_{j=1}^{N} y_j/N$. ∎

From the objective function of player i, one can see that as the number of players in a group increases, the incentive to free-ride on the others' efforts increases. When both the distance and the cost functions are linear so that the smaller group has no disadvantage in terms of the cost or the utility, any policy can be a stable outcome of the game because the larger group suffers more with free-rider problem than the smaller group does. In contrast, when the cost function is convex, while the asymmetry in the severity of free-rider problem still exists, the cost disadvantage breaks the balance between the groups. The balance can be recovered only when the relative powers between the groups are equalized.

This proposition shows that if the policy converges, it does to a "center" of which definition depends on the convexity of the distance measure (λ) and that of the cost function (γ). The convergence point can be the median, the mean of two extreme players, the mean of all players or some point between a mean and the median. An interesting question to ask is to which point the steady-state policy would converge (i) as λ goes to infinity, or (ii) as γ goes to infinity. First, if λ is very high, the marginal utility of having a policy closer to the ideal point will be extremely high for the players at the extremes compared to the other players'. Thus, as the distance measure becomes more convex (i.e., as $\left(|S - y_j|\right)^{\frac{\lambda-1}{\gamma-1}}$) becomes more sensitive to the distance), the steady-state policy would get closer to $1/2$, the middle point of the two extremes. It is not difficult to see that this result resonates with the case of $\gamma = 1$.

If the cost function is extremely convex, on the other hand, in the limit, the cost will be zero up to a certain point, then suddenly become infinity. Thus, it will be like everybody has one "vote" (observe that $\left(|S - y_j|\right)^{\frac{\lambda-1}{\gamma-1}}$ becomes 1 as γ goes to infinity), therefore the policy that the median voter prefers will be implemented. Of course, this result is comparable to the case with $\lambda = 1$.

What will steady-state equilibria look like if players are heterogeneous? Suppose first that $\gamma = 1$ and $\lambda = 1$. When the players are identical in their valuation, this free-riding incentive prevents the policy from converging to a center. In contrast, if the players are heterogeneous, everybody but those with the highest marginal utility free-rides completely. Thus, the policy would converge to a point between the ideal points of two players with the highest marginal utilities. This case is comparable to the model of Baik (2016) who analyzes a group contest where the players decide whether to support one or both of the two alternatives. In his model, the cost function is assumed to be linear, and the (asymmetric) marginal utilities are exogenously given.

Suppose that $\gamma > 1$ and $\lambda = 1$ and that players differ in their valuation (i.e., for some $i \neq j, \alpha_i \neq \alpha_j$) and in their power or resource (i.e., $\mu_i \neq \mu_j$).[7]

Define the power-adjusted valuation as $\bar{\alpha}_i = \left(\frac{\alpha_i}{\mu_i}\right)^{1/(\gamma-1)}$. If there exists a player i such that $\left|\sum_{j\in L(y_i)} \bar{\alpha}_j - \sum_{j\in R(y_i)} \bar{\alpha}_j\right| < \bar{\alpha}_j$, then it is straightforward to show that y_i can be a steady-state equilibrium policy. That is, such player i is a median influencer. Notice that the inequality holds more easily when $\bar{\alpha}_i$ is greater. So the steady-state outcome is likely to be the ideal policy of a strong player who is more or less in the middle. If there does not exist such a player, the steady-state outcome will be somewhere in between two median players. This type of "median voter theorem" (or some variations of it) has often been used by some political scientists to predict the outcome of a complicated political game (e.g. Bueno de Mesquita, 2000, 2002). Its performance has proven outstanding, but it is difficult to say that such practices have always

[7] Recall that μ_i is a factor multiplied to the effort cost. So, a smaller μ_i represents a stronger power.

been firmly micro-founded. The analysis in this subsection provides a micro-foundation of such forecasting exercises.

The third and fourth parts of Proposition 1 also can easily be generalized to the case with heterogeneous players. If $\gamma = 1$ and λ is sufficiently large, then still only the two extreme players will be active. However, the steady-state policy will not be the exact mean of the two ideal policies but a weighted average of which weight reflects the power-adjusted valuations of the two players.

Lastly, suppose that $\gamma = \lambda > 0$. Then, for $\sum_{j\in L} x_j^*$ to equal$\sum_{j\in R} x_j^*$, $\sum_{j\in L} \bar{\alpha}_j |S - y_j|$ must be equal to $\sum_{j\in R} \bar{\alpha}_j |S - y_j|$. Therefore, the steady-state outcome must be the weighted average of the players' ideal policies: $S = \frac{\sum_{j=1}^{N} \bar{\alpha}_j y_j}{\sum_{j=1}^{N} \bar{\alpha}_j}$.

4. Dynamic Analyses

In this Section we explore the dynamics of an infinite-horizon model to show that the incentive to free-ride slows down the convergence to the median player. First we characterize the dynamics under certain conditions. Then we run a robustness check.

4.1 Dynamic Equilibrium

In this subsection, we investigate the dynamics of the model, assuming that (i) the adjustment function is the Tullock-type CSF, (ii) $\lambda = 1$ and (iii) $N = 3$. Suppose that the game analyzed so far is repeated infinitely many times ($t = 1,2,..,\infty$) and that for any i and t, player i maximizes the discounted utility:

$$U_{it} = \sum_{\tau=t}^{\infty} \beta^{\tau-t}\, u_i(\delta_\tau, x_{i\tau}) = \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[-\alpha|\delta_\tau - y_i| - x_{i\tau}^{\gamma}/\gamma\right]$$

where $\beta \in [0,1)$ is the common discount factor, and x_{it} is the effort exerted at period t. The status quo at period $t+1$ is given by the implemented policy at t (i.e., $S_{t+1} = \delta_t$). For β sufficiently large, there exist infinitely many (collusive) subgame perfect equilibria which depend on the history of actions, which we do not intend to explore here. Instead, we focus on equilibria in which a strategy x_{it} is a function of the status quo S_t. If $\gamma = 1$, any policy can be a steady-state policy, which means that in such a case, the dynamics is either trivial or arbitrary. Thus, in this subsection, the cost function is assumed to be strictly convex.

Since the dynamics from the left to the right and the other way around are symmetric, we only consider the case with $S_1 < y_m = y_2$. As will be shown more clearly in the proof of Proposition 2, given $S_1 < y_m$, $\delta_t \leq y_m$ for all t. In other words, the equilibrium policy never crosses the ideal point of the median player. This implies that $\hat{L}_t = \{1\}$ and $\hat{R}_t = \{2, 3\}$ for all t.

Using the fact that $S_{t+1} = S_t + p_t = S_{t-1} + p_{t-1} + p_t = \ldots = S_{t-k} + \sum_{\tau=0}^{k} p_{t-\tau}$, we can rewrite the discounted utility as:

$$\sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[-\alpha|S_\tau + p_\tau - y_i| - x_{i\tau}^{\gamma}/\gamma\right]$$

$$= -\alpha \left|\frac{1}{1-\beta}(S_\tau + p_\tau - y_i) + \sum_{\tau=t+1}^{\infty} \beta^{\tau-t} \sum_{\eta=t+1}^{\tau} p_\eta \right| - \frac{x_{it}^{\gamma}}{\gamma} - \sum_{\tau=t+1}^{\infty} \beta^{\tau-t} \frac{x_{i\tau}^{\gamma}}{\gamma}$$

One can easily see that because a change in the policy has a permanent effect, the marginal utility of having a more desirable policy is always $\alpha/(1-\beta)$, meaning that it does not depend on S_t. Then by envelop theorem, we can ignore the decision in period $t+1$ and onward ($\{x_{i\tau}\}_{\tau+1}^{\infty}$) when considering the decision making at period t. Thus, the first-order condition with respect to x_{it} is:

$$\frac{\alpha}{1-\beta} \left| \frac{\partial p(x_{it}, \boldsymbol{x}_{-it}^*, \theta_t^*)}{\partial x_{it}} |_{x_{it} = x_{it}^*} \right| - (x_{it}^*)^{\gamma-1} \geq 0$$

where the inequality condition is for the median player facing S_t close enough to y_m.

To proceed further, suppose the adjustment function is the Tullock-type CSF defined in (2). Let us suppose for a moment that all the FOCs hold as equality as S_t is far enough from y_m. In this case, $x_{2t}^* = x_{3t}^*$ because player 2 and 3 are in the same group and because their FOCs are identical. Thus, the FOCs are:

$$\left(\frac{\alpha}{1-\beta}\right)\left(\frac{2x_{2t}^*}{(x_{1t}^* + 2x_{2t}^*)^2}\right) = (x_{1t}^*)^{\gamma} - 1 \qquad (5)$$

$$\left(\frac{\alpha}{1-\beta}\right)\left(\frac{2x_{1t}^*}{(x_{1t}^* + 2x_{1t}^*)^2}\right) = (x_{2t}^*)^{\gamma} - 1 \qquad (6)$$

from which we derive the following proposition.

Proposition 2. Suppose that the adjustment function is the Tullock-type CSF. For any initial policy $S_1 \in [0,1]$, there exists an equilibrium in which $\{\delta_t\}_{t=1}^{\infty}$ converges to the median player's ideal policy y_m. As α or β gets larger, the equilibrium effort level x_{it}^* also grows larger. However, the speed of convergence does not depend on α and β, but it increases in γ.

Proof. We prove the proposition by construction. Observe that according to (3), given the groups and the efforts x, S and δ are one-to-one. Thus, we can recover S from δ. Using this idea, an equilibrium can be constructed as follows. First, pick a dividing rule $\bar{\theta}_t(< y_m)$, and calculate the optimal efforts using (6) and (7). This yields the equilibrium efforts:

$$x_{1t}^* = 2^{\frac{1}{\gamma}}\left(\frac{\alpha}{1-\beta}\frac{2^{\frac{1}{\gamma}}}{2+2^{\frac{1}{\gamma}}}\right)^{\frac{1}{\gamma}}$$

$$x_{2t}^* = x_{3t}^* = \left(\frac{\alpha}{1-\beta}\frac{2^{\frac{1}{\gamma}}}{2+2^{\frac{1}{\gamma}}}\right)^{\frac{1}{\gamma}}$$

and the adjustment $p(x_t^*, \bar{\theta}_t) = \left(\frac{2-2^{\frac{1}{\gamma}}}{2+2^{\frac{1}{\gamma}}}\right) = \bar{p}$.

Let $\delta_t = \bar{\theta}_t$, and solve (3) for S_t. Denote this calculated status quo by $\bar{S}_t$. Then, given $\bar{S}_t$, an equilibrium policy is $\bar{\theta}_t$. Note that when $\bar{\theta} = y_m$, the first-order condition for the median player does not have to hold as equality. Let us define $S_{m+} = y_m + \bar{p}$ and $S_{m-} = y_m - \bar{p}$. Then, for $S_t \in [S_{m-}, S_{m+}]$, instead of the equality FOC of the median player, equation $p(x_m, \boldsymbol{x_{-m}}, d^*) = y_m - S_t$ together with the FOCs for the other players characterizes the equilibrium.

In this equilibrium, $\{\delta_t\}_{t=1}^{\infty}$ converges to the median in each round as much as $\bar{p}$ when $S_t \notin [S_{m-}, S_{m+}]$, and once $S_t \in [S_{m-}, S_{m+}]$, δ_t is decided to be y_m, and stays there forever. Thus, as claimed above, $\{\delta_t\}_{t=1}^{\infty}$ does not oscillate around y_m, so the initial grouping remains valid until δ_t reaches y_m. From the above formulas, one can easily see that x_{it}^* increases in α and β, and that the speed of convergence does not depend on α and β but on γ. More specifically, $\bar{p}$ increases in γ. ■

We have reasonable belief that $\{\delta_t\}_{t=1}^{\infty}$ converges to the median under a set of more relaxed assumptions, and the result regarding the equilibrium effort level will remain valid. However, the speed of convergence may depend on α and β if another type of adjustment function is used, if the players are asymmetric, or when the distance measure is non-linear.

It is also worth mentioning that the speed of convergence is determined by both the free-riding incentive and the cost advantage: the amount of efforts in the larger group $(x_{2t}^{*}+x_{3t}^{*})$ is not twice as large as the effort level in the smaller group (x_{1t}^{*}) because the players in the larger group have an incentive to free-ride on each other's effort, which slows down the convergence. As γ gets larger, on the other hand, the cost advantage of the larger group becomes more significant, so the policy converges faster to the median.

4.2 Robustness in dynamics

One may wonder whether our main results remain valid even if the functional form assumptions on the distance and the cost functions are relaxed. The answer is yes if we make a simplifying assumption on the adjustment function as follows:

$$p(\boldsymbol{x},\boldsymbol{d})=\eta\sum_{j=1}^{n}d_j x_j$$

for some η positive but small. That is, the adjustment function is linear. Then, we can show the following.

Proposition 3. Suppose the adjustment function is linear. Let us further assume that the distance function $\|\delta-y\|$ and the cost function $c_i(x)$ are strictly convex and continuously differentiable and that the first derivative of the distance function is zero at $\delta=y$, i.e., $\frac{\partial\|\delta-y\|}{\partial\delta|_{\delta=y}}=0$. Then, the steady state equilibrium exists and is unique. If the agents behave myopically (i.e., the static optimal behavior is repeated), the policy δ converges to the steady state equilibrium.

Proof. Given the status quo policy, an individual's maximization problem has an interior solution because the distance and the cost functions are strictly convex. And the first-order condition is

$$\alpha_i\eta\frac{\partial\|\delta-y_i\|}{\partial\delta}|_{\delta=\delta^*}-c_i'(x^*)=0$$

Let $x_i(\tilde{\delta})$ be the optimal effort given the policy $\tilde{\delta}$, i.e.,

$$x_i(\tilde{\delta})=(c')^{-1}\left(\alpha_i\eta\frac{\partial\|\delta-y_i\|}{\partial\delta}|_{\delta=\tilde{\delta}}\right).$$

Since $c_i(x)$ is strictly convex and continously differentiable, the inverse function of the marginal cost is monotone increasing and continuous. Therefore, the optimal effort increases as $\tilde{\delta}$ moves away from y_i. Define

$$f(\tilde{\delta}) = \sum_{i \in L(\tilde{\delta})} x_i(\tilde{\delta}) - \sum_{i \in R(\tilde{\delta})} x_i(\tilde{\delta}).$$

Note that $f(0) \leq 0 \leq f(1)$ because $y_i \in [0,1]$ for all i and that $f(\tilde{\delta})$ is increasing. $f(\tilde{\delta})$ continous in $\tilde{\delta}$ because $\frac{\partial \|\delta - y\|}{\partial \delta}|_{\delta=y} = 0$. Thus, $f(\tilde{\delta}) = 0$ for a $\tilde{\delta}$, and such $\tilde{\delta}$ is unique.

To see the myopic dynamics, suppose that the equilibrium policy δ^* is smaller than the steady state policy, which means that $f(\delta^*) < 0$ or equivalently $p(\boldsymbol{x}, \boldsymbol{d}) > 0$. Since the adjustment is made toward the right extreme, the status quo S must be even smaller than δ^*. In other words, S was farther from the steady state than δ^* is. Thus, the static equilibrium policy converges toward the steady state policy. The analysis for δ^* is smaller than the steady state policy is analogous, and thus omitted. ■

5. Discussion

We construct a spatial voting model without the "one person, one vote" restriction and show that in equilibrium two groups endogenously emerge who try to implement opposing policies. We also show that depending on costs and preferences, a central policy (e.g., median / mean / middle) is the steady state equilibrium. The results are summarized in Table 1 below.

Table 1. Result summary

Effort cost	Distance cost	Steady state Equilibrium
Linear	Linear	Any policy
Convex	Linear	Median player's ideal policy
Linear	Convex	Mean of two extreme players' ideal policy
Convex	Convex	Mean of all players' ideal policies

Proposition 1 and the table above essentially provide the *Central Influencer* results for the static case. It shows that if both the disutility of non-optimal policy and the effort cost function are linear, any policy can be a steady-state policy. However, if the disutility of non-optimal policy is linear and the effort cost function is convex, the optimal policy for a median player is a steady-state policy. This is comparable to the *standard median voter theorem*. If the disutility of non-optimal policy is convex and the effort cost function is linear, then our model is very similar to that of Baik (2017). Whereas Baik's model resembles a representative democracy, ours is comparable to a direct democracy. Unlike Baik (2017), the marginal utility of exerting effort is endogenous in our model, and hence we find that the steady-state policy is the mean of the two extreme players' optimal policies. When the disutility of non-optimal policy and the effort cost function are equally convex, the steady-state policy is the (weighted) average of all players' optimal policies. Duggan and Gao (2020), in another setting, found that 'risk loving equilibrium' is the mean of all players' ideal points. In our setting the players are risk neutral, but they form group endogenously and we reach to similar conclusions.

We implement a 3-player infinite horizon model to study dynamics. We find that in an infinite horizon model, there exists an equilibrium in which the policy converges to the median player, but in general it may not happen. We also find that the players expend more effort in each period as they become more patient. The convergence speed does not depend on the discount factor, but does depend on the effort cost. These results are in contrast with the dynamic models of Baron (1996) who finds that in collective goods programs equilibrium programs ultimately converge to the median.

Overall, parts of our results can be matched with Gerber and Lewis (2004) who find empirically that legislative decisions are aligned with median voter only under certain conditions. Our results can also be compared with a series of results that show in very different settings that median voter type of equilibrium may not be achieved. Overall, these result support the idea raised by Hinich (1977) that median voter results are often artefact instead of a general result.

Our results can have implications in the spatial competition model as well and connect to the researchers in lobbying and politics. This model has been the workhorse model in the domain of general interest politics but has not been very popular among the studies in special interest politics. As an exception, Coate (2004) considers a game in which politicians finance advertising campaign and interest groups provide contributions to like-minded candidates. But not all political influences are mediated by or aimed for a public election, e.g. international politics. We consider a direct competition among influential parties on a single-dimension policy space. Moreover, a line of literature in the political science have forecasted the outcome of complicated political games using a version of median voter theorem (e.g., Bueno de Mesquita, 2000, 2002). But it has been difficult to conclude that such practices have always been firmly micro-founded. We provide a micro-foundation for the same.

The study be advanced in to various directions. First, it would be interesting to generalize the model with generic cost and disutility functions, extending the policy space beyond linearity, and including more players in the dynamic version. Second, the collective action function or the network has been assumed to be additive (a la, Katz et al., 1990). Network effects such as Weakest link (Lee, 2012), best shot (Chowdhury et al., 2014), or a mix (Chowdhury and Topolyan, 2016) can be introduced. Finally, empirical and experimental investigations of the theoretical findings may be possible. None of these extensions, however, substitutes the new findings that we have achieved in this study and hence we leave these ideas for future research.

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