Inefficient Relative Price Fluctuations*

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Abstract

We measure inefficient fluctuations in the relative price of investment in the US using an estimated two-sector New Keynesian model. In the presence of these fluctuations, we find that monetary policy faces a quantitatively significant trade-off among the sectoral output gaps and the sectoral price and wage inflation rates. While optimal monetary policy is effective in stabilizing the sectoral inflation rates, it has a limited effect on stabilizing the sectoral output gaps.

Key Words: Relative price of investment, Policy trade-off, Optimal monetary policy
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1 Introduction

There is a growing interest in the extent to which changes in the relative price of investment accounts for aggregate fluctuations.[1] Existing studies typically assume that fluctuations in the relative price of investment are efficient in the sense that the relative price is determined under perfect competition (and flexible prices) in both the consumption and investment sectors. In particular, these studies attribute all of the observed fluctuations in the relative price of investment to exogenous changes in investment-specific technology (IST) by imposing a strict restriction that the period by period relative price of investment equals the inverse of the IST level.

However, recent studies suggest empirical evidence that the relative price of investment is distorted. Ikeda (2015) and Moura (2018) finds a high degree of stickiness in the relative price of investment, engendered from price stickiness in the consumption and investment sectors using the estimated model. Moreover, Moura (2018) finds that the sectoral price markup shocks drive the majority of variations in the relative price of investment at business cycle frequencies. Sticky prices and the sectoral price markup shocks drive a wedge between the IST and the observed relative price of investment. This wedge moves the economy away from its efficient frontier, calling for policy intervention. Given the empirical relevance of the distorted relative price of investment, the following questions arise. What is the measure that assesses the extent to which the relative price is distorted? What are the inefficiencies and policy trade-offs implied by the distorted relative price of investment?

To this end, we construct and estimate a two-sector New Keynesian model with consumption and investment-goods producing sectors. Our model allows the relative price of investment to fluctuate for several distinct reasons. First, in addition to IST shocks, we allow other disturbances to affect the efficient relative price of investment. To do so, we allow for the case in which factor prices are not equal between the two sectors because of limited inter-sectoral factor mobility. Because different factor prices across sectors result in different nominal marginal costs, the relative price of investment can be *a priori* affected by aggregate shocks such as the aggregate total factor productivity (TFP), marginal efficiency of investment (MEI) and preference shocks even when both sectors have perfectly competitive markets and flexible prices.[2] Second, we do not restrict ourselves to symmetric price and nominal wage rigid-

[1] Examples include Greenwood, Hercowitz and Krusell (2000), Fisher (2006), Justiniano, Primiceri and Tambalotti (2011), and Guerrieri, Henderson and Kim (2014).

[2] Following Justiniano, Primiceri and Tambalotti (2011) and Moura (2018), we make a distinction between IST and MEI shocks in our model. IST shock is a productivity shock specific to the investment-goods

ity between the two sectors and allow for heterogeneity in the degree of price and nominal wage stickiness. Third, we introduce exogenous variations in sectoral market competitiveness. Hence, the observed relative price of investment diverges from its efficient level due to different time-varying markups across sectors that arise from sectoral markup shocks as well as the stickiness of prices and wages in our model.

Using our estimated model, we uncover the inefficient component of the relative price movements by measuring the distance between the actual and efficient level of the relative price. We define this distance as the *relative price gap*. Furthermore, we estimate the extent of inefficient fluctuations induced by the relative price gap. The inefficient variations in output in a particular sector are uncovered by measuring the difference between actual and efficient output, which we define as the output level that would be observed under perfect competition and thus zero markups, in that sector. We define this difference as the *sectoral output gap*.

We show that the relative price gap substantially fluctuates around its mean. The variations in the relative price gap are associated with sizable fluctuations in the two sectoral output gaps and the negative comovement between the two. Also, we find a significant degree of sectoral price and wage stickiness in our estimated model. As a result, variations in the sectoral price and wage inflation rates constitute additional sources of inefficiencies. Therefore, in general, the monetary authority confronts a complex trade-off among the stabilization of the sectoral output gaps, the sectoral price inflation rates, and the sectoral wage inflation rates in our economy.

To evaluate the significance of this trade-off, we compute the evolution of the counterfactual economy that would be observed if a central bank follows the optimal monetary policy, the interest rate path that maximizes the representative household utility, rather than the interest rule we estimate. We find that the welfare gains from the optimal monetary policy mostly stem from a stabilization of sectoral inflation, in particular, wage inflation. However, much of the fluctuations in the sectoral output gaps remain due to the significant degree of monetary policy trade-off. This finding contrasts with that of Justiniano, Primiceri and Tambalotti (2013), who show that optimal monetary policy is sufficient to achieve joint stabilization of the output gap and inflation. They draw out such a conclusion using the estimated one-sector New Keynesian model, in which the relative price gaps are absent by construction. Using

producing sector. MEI is a shock to the process by which these investment goods are turned into future productive capital input. According to Justiniano, Primiceri and Tambalotti (2011), the MEI shock is interpreted as a disturbance to the financial system's ability to provide funds to firms, orthogonal to the productivity shock in the investment sector.

our counterfactual two-sector model in which the relative price gaps are zero, we obtain a similar result to theirs, implying that the distorted relative price of investment is the key to the monetary policy trade-off.

The sources behind the lack of joint stabilization of the sectoral output gaps and inflation under the optimal monetary policy are the IST and sectoral price markup shocks. Each of these shocks causes a different kind of trade-off. IST shocks induce a trade-off between stabilizing the output gaps across sectors, making it difficult for the monetary authority to stabilize both output gaps at the same time. A sector-specific price markup shock induces not only a trade-off between stabilizing the output gaps across sectors but also a trade-off between stabilizing the output gap and inflation within each sector. The latter involves the volatile output gap in any given sector, which is the price that the monetary authority has to pay to reduce the volatility of inflation in that sector.

Previous papers have also investigated the optimal conduct of monetary policy in a multi-sector or multi-country model. Important works include Aoki (2001), Benigno (2004), Erceg and Levin (2006), Barsky et al. (2016), and Basu and De Leo (2019). These papers emphasize the presence of a trade-off when relative prices are distorted in a calibrated or theoretical model that abstracts from one or more key ingredients that are essential in determining the extent to which relative prices are distorted. These key ingredients are sticky prices, sticky wages, and sectoral markup shocks. In contrast, we provide an empirical measure of the extent to which relative prices are distorted using a model that embeds all these key ingredients, which are estimated. We demonstrate that the monetary policy trade-off is quantitatively significant.

This paper is also related to the literature on the estimation of two-sector DSGE models with consumption and investment goods-producing sectors (Moura, 2018; Ikeda, 2015; Katayama and Kim, 2018). Moura (2018) focuses on positive analysis: the empirical relevance of sticky relative price of investment and its implication on the sources of economic fluctuations. In contrast, our focus is on the normative implication of the distorted relative price of investment. Ikeda (2015) focuses on deriving the steady state optimal consumption sector inflation rate. However, we keep the state steady inflation fixed and characterize allocations under the Ramsey optimal monetary policy. Katayama and Kim (2018) study sectoral and aggregate comovement issues using a model with perfectly competitive goods and labor markets.

The rest of our paper is organized as follows. Section 2 presents our two-sector New Keynesian model in which business cycles are driven by both sectoral and aggregate shocks. In section 3, we define the relative price gap and investigate its determinants analytically. Section 4 describes the data and the estimation procedure. In section 5, we compare inefficient fluctuations under the estimated and optimal monetary policy. Section 6 explores why joint stabilization of the sectoral output gaps and inflation do not emerge under the optimal monetary policy. Section 7 discusses the choice of shocks and practical implications of our results for policymaking. Finally, our conclusion is laid out in Section 8.

2 Model

The model is similar to the two-sector New Keynesian model with limited factor mobility presented in Moura (2018). A notable feature of our model is that there are shocks that are not present in his model. In section 7.1, we discuss the extent to which these shocks matter for normative results. Given that many ingredients we introduce are present in Moura (2018), we keep the description of the model short.

2.1 Firms

There are two sectors that produce sector-specific output, namely, consumption and investment goods. We use the subscript $j \in \{c, i\}$ to refer the sector, where c and i correspond to the consumption and investment sector, respectively. In sector j, a representative final good firm combines a continuum of intermediate goods $\{Y_{j,t}(s)\}_s$, $s \in [0, 1]$, and produces the sectoral output $Y_{j,t}$ according to the technology

$$Y_{j,t} = \left[\int_0^1 Y_{j,t}(s)^{\frac{1}{1+\eta^p_{j,t}}} ds \right]^{1+\eta^p_{j,t}},$$

where $\eta^p_{j,t} > 0$ denotes the sector-specific markup in the market for intermediate goods and evolves according to

$$\log(1 + \eta^p_{j,t}) = (1 - \rho_{\eta^p_j})\log(1 + \eta^p) + \rho_{\eta^p_j}\log(1 + \eta^p_{j,t-1}) + \epsilon_t^{\eta^p_j}, \qquad \epsilon_t^{\eta^p_j} \overset{iid}{\sim} \mathcal{N}(0, \sigma^2_{\eta^p_j}),$$

where η^p is the steady state price markup. Within each sector, a monopolistically competitive firm produces intermediate goods using capital and labor services. All firms within each sector use identical technology, represented by the production function

$$Y_{c,t}(s) = K_{c,t}(s)^\alpha (A_t z_t N_{c,t}(s))^{1-\alpha} - \Omega_{c,t} F_c,$$
$$Y_{i,t}(s) = K_{i,t}(s)^\alpha (A_t z_t z_{i,t} N_{i,t}(s))^{1-\alpha} - \Omega_{i,t} F_i,$$

where $K_{j,t}(s)$ and $N_{j,t}(s)$ denote capital and labor services used by intermediate good producing firm s in sector j. α and F_j measure the capital share and the fixed cost in sector j, while $\Omega_{j,t}$ is a stochastic trend which ensures proper scaling of the fixed cost. $z_{i,t}$ is the investment-specific technology (IST), and its growth rate $\mu_{j,t} = \frac{z_{j,t}}{z_{j,t-1}}$ evolves according to

$$\log\mu_{i,t} = (1-\rho_{\mu_i})\log\mu_i + \rho_{\mu_i}\log\mu_{i,t-1} + \epsilon_t^{\mu_i}, \qquad \epsilon_t^{\mu_i} \overset{iid}{\sim} \mathcal{N}(0, \sigma_{\mu_i}^2),$$

where μ_i is the steady state growth rate of the IST. z_t is the non-stationary aggregate total factor productivity (TFP), and its growth rate $\mu_t^z = \frac{z_t}{z_{t-1}}$ evolves according to

$$\log\mu_t^z \;=\; (1-\rho_{\mu^z})\log\mu^z + \rho_{\mu^z}\log\mu_{t-1}^z + \epsilon_t^{\mu^z}, \qquad \epsilon_t^{\mu^z} \overset{iid}{\sim} \mathcal{N}(0, \sigma_{\mu^z}^2),$$

where μ^z is the steady state growth rate of the non-stationary aggregate TFP. A_t is the stationary aggregate total factor productivity and evolves according to

$$\log A_t \;=\; \rho_A \log A_{t-1} + \epsilon_t^A, \qquad \epsilon_t^A, \overset{iid}{\sim} \mathcal{N}(0, \sigma_A^2).$$

Intermediate good producing firms in each sector are subject to nominal price rigidity á la Calvo (1983). We let ξ_{pj} and ι_{pj} be the price stickiness and the price indexation parameter in sector j, respectively.

2.2 Households

The representative household maximizes the utility function

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \zeta_t \left(\frac{(C_t - hC_{t-1})^{1-\sigma} - 1}{1-\sigma} - b_t \frac{N_t^{1+\varphi}}{1+\varphi} \right),$$

where $\beta \in (0,1)$ is the subjective discount factor, $h \in (0,1)$ is the degree of habit formation, σ is the risk-aversion coefficient, and φ is the inverse of the Frisch elasticity of labor supply. b_t is the relative disutility of supplying aggregate labor and follows

$$\log b_t = (1-\rho_b)\log b + \rho_b \log b_{t-1} + \epsilon_t^b, \qquad \epsilon_t^b \overset{iid}{\sim} \mathcal{N}(0, \sigma_b^2).$$

The disturbance to the disutility of aggregate labor is labelled as the aggregate labor supply shock. As in Justiniano, Primiceri and Tambalotti (2013), we introduce this shock to produce a more realistic model-based GDP gap. In our model, this shock captures the cyclical movements of the common component of sectoral hours worked.

C_t is the period t consumption, and $N_t = [N_{c,t}^{1+\omega^N} + N_{i,t}^{1+\omega^N}]^{\frac{1}{1+\omega^N}}$ denotes aggregate hours worked, where ω^N measures the labor reallocation cost across sectors, as in Huffman and Wynne (1999) and Horvath (2000). ζ_t is an intertemporal preference shock that evolves according to

$$\log\zeta_t = \rho_\zeta \log\zeta_{t-1} + \epsilon_t^\zeta, \qquad \epsilon_t^\zeta \overset{iid}{\sim} \mathcal{N}(0, \sigma_\zeta^2).$$

The period t household budget constraint is given by

$$C_t + Q_t I_t + \frac{B_t}{P_{c,t}} + \frac{T_t}{P_{c,t}} \leq \frac{R_{t-1}B_{t-1}}{P_{c,t}} + \sum_{j=c,i}(1-\tau^n)\frac{\overline{W}_{j,t}N_{j,t}}{P_{c,t}} + Q_t \sum_{j=c,i}(1-\tau^k) r_{j,t}^k K_{j,t}$$
$$+ \frac{D_t}{P_{c,t}} - Q_t \Psi(u_t)\overline{K}_{t-1},$$

where $P_{j,t}$ is the price index in sector j. $Q_t = \frac{P_{i,t}}{P_{c,t}}$ is the relative price of investment goods. $\frac{\overline{W}_{j,t}N_{j,t}}{P_{c,t}}$ is real labor income from working in sector j and I_t denotes units of investment goods purchased. B_t represents the nominal value of a one-period government bond purchased in period t, R_t is the gross nominal interest rate, $\frac{D_t}{P_{c,t}}$ is the sum of real profits rebated by firms and labor unions, and T_t is the lump-sum tax paid to the government.

The household owns capital and chooses the capital utilization rate u_t which transforms aggregate physical capital into effective aggregate capital according to $K_t = u_t \overline{K}_{t-1}$. The cost of capital utilization is $\Psi(u_t)$ per unit of physical capital. We parameterize it as $\Psi(u_t) = \rho^u \frac{u_t^{\frac{1}{1-\psi}} - 1}{\frac{1}{1-\psi}}$ such that in steady state, $u = 1$, $\Psi(1) = 0$ and $\frac{\Psi''(1)}{\Psi'(1)} = \frac{\psi}{1-\psi}$, where $\psi \in (0,1)$. To incorporate limited capital mobility, as in Moura (2018), we use a specification similar to that of aggregate hours worked. Namely, effective aggregate capital in period t is

$$K_t = [K_{c,t}^{1+\omega^K} + K_{i,t}^{1+\omega^K}]^{\frac{1}{1+\omega^K}},$$

where $\omega^K \geq 0$ captures the capital reallocation cost across sectors. Sector j effective capital $K_{j,t}$ is then rented to firms in sector j, and the household receives rental income $Q_t r_{j,t}^k K_{j,t}$, where $r_{j,t}^k \equiv \frac{R_{j,t}^k}{P_{i,t}}$. τ^n and τ^k are the labor income tax rate and the capital income tax rate, respectively. Following the convention in the optimal monetary policy literature, we fix these tax rates at constant values that ensure efficient allocation at the steady state.[3]

Aggregate physical capital accumulates according to

$$\overline{K}_t = \upsilon_t I_t \left[1 - \frac{\kappa}{2}\left(\frac{I_t}{I_{t-1}} - \mu_i\right)^2\right] + (1-\delta)\overline{K}_{t-1},$$

where δ denotes the depreciation rate, and κ captures the convex investment adjustment cost proposed by Christiano, Eichenbaum and Evans (2005). υ_t is the marginal efficiency of investment (MEI), which evolves according to

$$\log \upsilon_t = \rho_\upsilon \log \upsilon_{t-1} + \epsilon_t^\upsilon, \qquad \epsilon_t^\upsilon \overset{iid}{\sim} \mathcal{N}(0, \sigma_\upsilon^2).$$

Finally, the household supplies hours worked to sector-specific labor unions. As in Erceg, Henderson and Levin (2000), in each sector, labor unions differentiate homogeneous hours worked and set nominal wages subject to Calvo pricing frictions. We let ξ_{wj} and ι_{wj} be the wage stickiness and the wage indexation parameters in sector j, respectively. Competitive labor packers combine a continuum of differentiated labor services $\{N_{j,t}(\tilde{s})\}_{\tilde{s}}$, $\tilde{s} \in [0,1]$, and produce the usable labor input according to the technology

$$N_{j,t} = \left(\int_0^1 N_{j,t}(\tilde{s})^{\frac{1}{1+\eta_{j,t}^w}} d\tilde{s}\right)^{1+\eta_{j,t}^w}.$$

$\eta_{j,t}^w > 0$ is the sector-specific wage markup and evolves according to

$$\log(1+\eta_{j,t}^w) = (1-\rho_{\eta_j^w})\log(1+\eta^w) + \rho_{\eta_j^w}\log(1+\eta_{j,t-1}^w) + \epsilon_t^{\eta_j^w}, \qquad \epsilon_t^{\eta_j^w} \overset{iid}{\sim} \mathcal{N}(0, \sigma_{\eta_j^w}^2),$$

where η^w is the steady state wage markup. Sector-specific wage markup shocks capture the deviation of sector-specific hours from the common hours.

2.3 Government

We assume the monetary authority sets the policy rate in a way that reacts to the previous nominal interest rate, deviations of annual inflation from a time-varying inflation target, and

[3] Specifically, $\tau^n = 1 - (1+\eta^p)(1+\eta^w)$, and $\tau^k = -\eta^p$.

deviations of observed annual GDP growth from its steady state level

$$\log\left(\frac{R_t}{R}\right) = \rho_R \log\left(\frac{R_{t-1}}{R}\right) + (1-\rho_R)[\phi_\pi\left(\frac{1}{4}\sum_{\iota=0}^{3}\log\left(\frac{\pi_{c,t-\iota}}{\pi^*_{c,t}}\right)\right)$$
$$+\phi_x\left(\frac{1}{4}\sum_{\iota=0}^{3}\log\left(\frac{X_{t-\iota}}{\mu X_{t-1-\iota}}\right)\right)] + \epsilon^m_t, \tag{1}$$

where ϵ^m_t captures a monetary policy shock with $\epsilon^m_t \overset{iid}{\sim} \mathcal{N}(0,\sigma^2_m)$. X_t is real GDP in consumption units. The inflation target evolves exogenously according to the process

$$\log\pi^*_{c,t} = (1-\rho_{\pi_c})\log\pi_c + \rho_{\pi_c}\log\pi^*_{c,t-1} + \epsilon^{\pi_c}_t, \qquad \epsilon^{\pi_c}_t \overset{iid}{\sim} \mathcal{N}(0,\sigma^2_{\pi_c}).$$

The government purchases exogenous amounts of consumption and investment goods, respectively denoted by $G_{c,t}$ and $G_{i,t}$. Letting $g_{c,t} = \frac{G_{c,t}}{\Omega_{c,t}}$ and $g_{i,t} = \frac{G_{i,t}}{\Omega_{i,t}}$, we assume that

$$\log g_{c,t} = (1-\rho_{g_c})\log g_c + \rho_{g_c}\log g_{c,t-1} + \epsilon^{g_c}_t, \qquad \epsilon^{g_c}_t \overset{iid}{\sim} \mathcal{N}(0,\sigma^2_{g_c})$$
$$\log g_{i,t} = (1-\rho_{g_i})\log g_i + \rho_{g_i}\log g_{i,t-1} + \epsilon^{g_i}_t, \qquad \epsilon^{g_i}_t \overset{iid}{\sim} \mathcal{N}(0,\sigma^2_{g_i}).$$

Lump-sum taxes adjust to balance the government budget constraint at each period.

3 The Relative Price Gap

As mentioned in the introduction, the extent of inefficient relative price movements is summarized by the movements in the relative price gap. Before we take our model to the data, we define the relative price gap and study its determinants analytically. Due to the different degree of imperfect competition in goods and labor markets across sectors and limited factor mobility, the relative price of investment is given as

$$Q_t = \frac{\mathcal{M}^p_{i,t} MC_{i,t}}{\mathcal{M}^p_{c,t} MC_{c,t}} = \frac{\mathcal{M}^p_{i,t}}{\mathcal{M}^p_{c,t}}\frac{1}{z^{1-\alpha}_{i,t}}\left(\frac{W_{i,t}}{W_{c,t}}\right)^{1-\alpha}\left(\frac{R^k_{i,t}}{R^k_{c,t}}\right)^{\alpha} = \frac{\mathcal{M}^p_{i,t}}{\mathcal{M}^p_{c,t}}\frac{1}{z^{1-\alpha}_{i,t}}\left(\frac{\mathcal{M}^w_{i,t}\overline{W}_{i,t}}{\mathcal{M}^w_{c,t}\overline{W}_{c,t}}\right)^{1-\alpha}\left(\frac{R^k_{i,t}}{R^k_{c,t}}\right)^{\alpha}, \tag{2}$$

where $\mathcal{M}^p_{j,t}$ and $\mathcal{M}^w_{j,t}$ are gross price and wage markup in sector j, respectively. $W_{j,t}$ denotes the price of one unit of labor that intermediate good producing firms in sector j have to pay. The efficient relative price, the relative price that would prevail under zero price and wage markups, is

$$Q_t^e = \frac{1}{z_{i,t}^{1-\alpha}} \left(\frac{\overline{W}_{i,t}^e}{\overline{W}_{c,t}^e} \right)^{1-\alpha} \left(\frac{R_{i,t}^{ke}}{R_{c,t}^{ke}} \right)^{\alpha}, \tag{3}$$

where the variable with the superscript e denotes the efficient level of that variable. A limited factor mobility leads to different responses of factor prices across sectors to any shocks, and so the tight link between the efficient relative price and the IST shock breaks. Then the relative price gap is defined as

$$\log(Q_t/Q_t^e) = \log \left(\frac{\mathcal{M}_{i,t}^p}{\mathcal{M}_{c,t}^p} \left(\frac{\mathcal{M}_{i,t}^w}{\mathcal{M}_{c,t}^w} \right)^{1-\alpha} \left(\frac{\overline{W}_{i,t}/\overline{W}_{c,t}}{\overline{W}_{i,t}^e/\overline{W}_{c,t}^e} \right)^{1-\alpha} \left(\frac{R_{i,t}^k/R_{c,t}^k}{R_{i,t}^{ke}/R_{c,t}^{ke}} \right)^{\alpha} \right). \tag{4}$$

This expression shows that changes in the relative price gap occur from three sources: shifts in relative price markups, shift in relative wage markups, and shifts in relative factor prices from their efficient level.

In the case of free factor mobility, factor prices are equal across sectors ($\overline{W}_{c,t} = \overline{W}_{i,t}$, $\overline{W}_{c,t}^e = \overline{W}_{i,t}^e$, $R_{c,t}^k = R_{i,t}^k$, $R_{c,t}^{ke} = R_{i,t}^{ke}$), and so only relative price markups and relative wage markups affect the relative price gap. For explanatory purposes, we focus on a case in which labor markets are perfectly competitive. When prices are flexible in both sectors, only sectoral price markup shocks shift the relative price gap. When prices are equally sticky, sectoral price markup and the IST shocks shift the relative price gap through a change in relative price markups. The rest of the shocks do not affect the relative price gap in this environment as these shocks lead to an equal response of nominal marginal costs across sectors and thus no change in relative price markups. When price stickiness differs across sectors, not only sectoral price markup and the IST shocks but also the remaining shocks affect the relative price gap through a change in relative price markups. The reason is that even though the nominal marginal costs are the same across sectors after non-price markup and non-IST shocks, different price stickiness implies different average markups, changing the relative price gap.[4] A similar logic carries out for the relative wage markups. That is, the relative wage markups depend on nominal wage stickiness in both sectors and the type of shocks.

In the case of limited factor mobility, the relative price gap depends on relative price markups, relative wage markups, and the distance between relative factor prices and their efficient level.

[4] Aoki (2001) shows that when one sector features flexible prices, and the other sector features sticky prices, the relative price gap is closed under the optimal monetary policy. However, such an allocation is not attained under a suboptimal monetary policy such as our monetary policy rule (1).

Again, to illustrate, assume a perfectly competitive labor market. When prices are flexible in both sectors, only sectoral price markup shocks shift the relative price markups and thus the relative price gap. Non-price markup shocks do not change the relative price gap in this environment as relative factor prices are at their efficient level after these shocks. When prices are equally sticky, unlike the case of free factor mobility, all shocks shift the relative price gap through a change in relative price markups. This is because any shock changes relative factor prices, and thus, nominal marginal costs respond differently across sectors. As a result, relative price markups change and thus, the relative price gap changes. When price stickiness differs across sectors, as in the case of free factor mobility, all shocks affect relative price markups and thus the relative price gap. However, the relative price markups not only depend on the extent to which price stickiness differs across sectors but also on the heterogeneous response of nominal marginal costs due to shifts in relative factor prices.

The one-sector model is a special case of our model and can be obtained by the following parameterizations: equally sticky prices, equally sticky nominal wages, symmetric price markup shocks, symmetric wage markup shocks, symmetric government spending shocks, no IST shock, and free factor mobility. Then relative price markups, relative wage markups, and relative factor prices are equal to 1, and thus the relative price of investment is equal to 1. Therefore, in a standard one-sector model such as Smets and Wouters (2007) and Justiniano, Primiceri and Tambalotti (2010), there are no inefficiencies associated with relative price fluctuations.

In sum, the ingredients that determine the volatility of the relative price gap are price and nominal wage stickiness in both sectors, the size of shocks, and the degree of factor mobility. In the next section, we estimate these ingredients to uncover the empirical relative price gap.

4 Model Solution and Estimation

We log-linearize the model's equilibrium conditions around the non-stochastic steady state and solve the resulting linear system using standard methods. We estimate the model's parameters using Bayesian methods.

Data We estimate the model using eleven observables: real private consumption growth, real private investment growth, real government consumption growth, real government investment growth, the logarithm of hours worked and the real wage growth in the consumption sector, the logarithm of hours worked and the real wage growth in the investment sector,

Table 1. Prior and posterior distribution

Parameter	Description	Prior dist.			Posterior dist.		
		Distribution	Mean	SD	Mean	5%	95%
h	Consumption habits	Beta	0.6	0.1	0.87	0.85	0.90
φ	Inv. Frisch elasticity	Gamma	2	0.75	0.95	0.64	1.27
ω^N	Labor reallocation cost	Gamma	2	0.75	0.17	0.07	0.26
ω^K	Capital reallocation cost	Gamma	2	0.75	0.15	0.06	0.24
κ	Invest. adjustment cost	Gamma	4	1	5.00	3.68	6.33
ψ	Capital utilization cost	Beta	0.5	0.15	0.94	0.89	0.98
ξ_{pc}	Calvo: C price	Beta	0.66	0.1	0.81	0.78	0.85
ι_{pc}	Indexation: C price	Beta	0.5	0.15	0.14	0.05	0.22
ξ_{pi}	Calvo: I price	Beta	0.66	0.1	0.87	0.83	0.90
ι_{pi}	Indexation: I price	Beta	0.5	0.15	0.10	0.04	0.17
ξ_{wc}	Calvo: C wage	Beta	0.66	0.1	0.84	0.80	0.87
ι_{wc}	Indexation: C wage	Beta	0.5	0.15	0.26	0.11	0.39
ξ_{wi}	Calvo: I wage	Beta	0.66	0.1	0.79	0.75	0.83
ι_{wi}	Indexation: I wage	Beta	0.5	0.15	0.24	0.11	0.36
ρ_R	Taylor rule: Smoothing	Beta	0.6	0.2	0.68	0.61	0.75
ϕ_π	Taylor rule: Inflation	Norm	1.7	0.3	3.17	2.86	3.53
ϕ_x	Taylor rule: Output	Norm	0.4	0.3	0.42	0.28	0.56
$100\sigma_{\eta_{wc}}$	Std C wage markup	Inv. Gamma	0.05	0.03	0.03	0.02	0.05
$100\sigma_{\eta_{wi}}$	Std I wage markup	Inv. Gamma	0.05	0.03	0.05	0.02	0.08
$100\sigma_{e_{wc}}$	Std C wage m. error	Inv. Gamma	0.5	1	0.24	0.21	0.27
$100\sigma_{e_{wi}}$	Std I wage m. error	Inv. Gamma	0.5	1	0.18	0.14	0.21

consumption price inflation, the relative price of investment growth, and the Federal funds rate. Private consumption corresponds to the sum of non-durables and services, while private investment is constructed by adding consumer durables to total private investment. To construct the sectoral series for hours and wages, it is necessary to classify the available series from the BLS to either the consumption or investment sector. We adopt the procedure used in Moura (2018). For constructing total investment hours, we sum the total hours worked in construction, durable manufacturing, and professional and business services. For total consumption hours, we take the difference between total private hours worked and total investment hours. For sectoral wages and other variables, we provide more details in Appendix A. Our sample starts from 1965Q1 due to the limited availability of the wage data and ends in 2008Q3, which is the quarter right before the nominal interest rate hit the zero lower bound in the US. All series are demeaned before estimation.

Prior and the measurement errors The following set of parameters are fixed during estimation. The risk-aversion coefficient σ is 1. The household discount factor β, the depreciation rate δ, the capital share α, and the steady state markups η^p and η^w are fixed at 0.999, 0.025, 0.3, and 0.25, respectively. We calibrate π_c, μ_z, and μ_i by matching the sample averages of the consumption price inflation rates, real private consumption growth, and real private investment growth. The steady state ratio of government consumption to private consumption and the ratio of government investment to private investment are set to their sample averages. Finally, the autocorrelation of the inflation target shock is fixed at 0.995 following Justiniano, Primiceri and Tambalotti (2013). The remaining parameters are estimated from the data.

We assume that our data series on the real wage imperfectly match the model's concept of the real wage due to well-known difficulties in measuring aggregate nominal wages (Boivin and Giannoni, 2006; Justiniano, Primiceri and Tambalotti, 2013; Eusepi, Giannoni and Preston, 2018). Accordingly, we augment the sectoral wage inflation rates in the model with sectoral measurement errors to match our data series on the sectoral real wages.[5] We impose a tight prior on the standard deviations of sectoral wage markup shocks and impose a wide prior for the standard deviations of sectoral measurement errors. Specifically, we use the posterior estimates of the standard deviations of wage markup shocks from Justiniano, Primiceri and Tambalotti (2013) for our priors on the standard deviations of sectoral wage markup shocks. As understood from equation (4), sectoral wage markup shocks can change the relative price gap. By suppressing the explanatory power of sectoral wage markup shocks on the sectoral real wages, we obtain relative price gap fluctuations that are mostly unrelated to sectoral wage markup shocks. In this sense, our estimate of the relative price gap is conservative.

The priors on the other parameters are fairly diffuse and broadly in line with those adopted in previous studies, such as Justiniano, Primiceri and Tambalotti (2013) and Moura (2018). Table 1 reports the prior and posterior distributions of the model's structural parameters. The results on the parameters of the remaining shock processes are reported in Appendix C.

Posterior estimates The posterior estimates of the Calvo parameters signal price and nominal wage stickiness in both sectors are quite similar, with mean values of $\xi_{pc} = 0.81$,

[5] Boivin and Giannoni (2006), Justiniano, Primiceri and Tambalotti (2013), and Eusepi, Giannoni and Preston (2018) link two wage series – nominal compensation per hour and average hourly earnings – with the model's wage and two measurement errors. Because only average hourly earnings are available at the sector-level, we adopt one wage series and one measurement error approach.

Table 2. Posterior variance decomposition at business cycle frequencies

Shock/series	GDP	C	I	Rel. price	C gap	I gap	RP gap
C price markup	19.47	13.36	9.17	17.77	65.34	28.94	10.50
I price markup	1.67	3.74	5.43	61.34	23.26	23.59	31.93
Aggregate TFP growth	11.31	13.40	3.46	0.45	0.49	0.27	0.02
IST growth	0.38	0.62	1.74	19.98	5.64	29.75	52.58
Aggregate TFP	2.59	3.64	1.64	0.23	0.07	0.15	0.00
MEI	58.13	6.12	75.45	0.09	1.22	12.32	4.53
Preference	0.93	50.64	0.10	0.02	0.04	0.00	0.07
C wage markup	0.03	0.02	0.01	0.03	0.09	0.04	0.01
I wage markup	0.00	0.00	0.00	0.02	0.01	0.03	0.02
Mon. policy	0.41	0.15	0.42	0.00	0.35	0.85	0.00
Gov. consumption	0.51	0.23	0.04	0.01	0.01	0.01	0.03
Gov. investment	0.71	0.21	0.03	0.00	0.01	0.02	0.05
Labor supply	3.78	7.73	2.42	0.06	3.39	3.84	0.26
Inflation target	0.09	0.04	0.09	0.00	0.09	0.19	0.00

Note: Decomposition computed at the posterior mean using the Band-Pass filter with the passband of 6-32 quarters. GDP, consumption (C), investment (I), and the relative price of investment (Rel. price) are in growth, and the consumption gap, investment gap, and relative price gap (RP gap) are in levels.

$\xi_{pi} = 0.87$, $\xi_{wc} = 0.84$ and $\xi_{wi} = 0.79$. Despite using the data on sectoral hours, the model favors small labor reallocation costs with a posterior mean value below the values estimated by Moura (2018) and Katayama and Kim (2018). In their model, some degree of labor immobility is needed to help the model explain a positive comovement of sectoral hours worked. Their relatively high estimates of labor reallocation costs emerge because of the absence of the aggregate labor supply shock. The aggregate labor supply shock in our model shifts the marginal rate of substitution between consumption and leisure in both sectors to the same degree, and hence affect the hours worked in both sectors uniformly. As a result, a large part of the comovement of sectoral hours worked are captured by this shock, and hence there is less need to rely on the labor immobility. The capital reallocation costs are small, consistent with Moura (2018). Overall, the data suggests that there is not much sectoral heterogeneity in nominal rigidity, and factor reallocation costs are small.

As expected from our prior structure, posterior estimates of the standard deviation of wage measurement errors are much larger than those of the standard deviation of the sectoral wage markup shocks, consistent with the results of Justiniano, Primiceri and Tambalotti (2013).[6]

[6] To check whether our results are robust to assuming less restrictive priors for the wage markup shocks, we have re-estimated the model imposing wide priors for the sectoral wage markup shocks (Inverse Gamma distribution with mean 0.15 and the standard deviation 1). We found that the posterior estimates are similar to the ones obtained from the baseline model, implying that the positive and normative results are barely

These estimates imply a small explanatory power of the sectoral wage markup shocks, which Chari, Kehoe and McGrattan (2009) label as dubiously structural shocks, for sectoral real wage growth, and thus policy trade-offs induced by sectoral wage markup shocks is quantitatively small.[7]

Table 2 reports the variance decomposition at business-cycle frequencies for six key variables: GDP in consumption units, consumption, investment, consumption hours, investment hours, and the relative price of investment. It is worth mentioning that our results are in line with earlier studies. The consumption and investment price markup shocks account for most of the volatility of the relative price of investment, while the contribution of the IST shock is relatively small. This finding confirms the results of Moura (2018), who shows that the conventional view of the IST shock playing a major role in explaining the relative price of investment does not hold. Moreover, the IST shock has a negligible role in explaining aggregate demand, consistent with the findings of Justiniano, Primiceri and Tambalotti (2011).

However, the small role of the IST shock in explaining the observable variables does not mean that policymakers should ignore this shock. What matters for a policy prescription is not how much IST shocks explain the observables but the volatility of the sectoral output gaps and the relative price gap caused by these shocks. As discussed in section 3, a significant degree of sticky prices and sticky nominal wages in our model can lead to a large role of the IST shock on the fluctuations in the relative price gap under the estimated monetary policy, generating a large degree of inefficient sectoral output fluctuations in our economy. In particular, IST shocks account for 53% of the cyclical variance of the relative price gap, 6% of that of the consumption gap, and 30% of that of the investment gap under the estimated monetary policy, as seen in Table 2. Therefore, IST shocks along with consumption and investment price markup shocks account for most of the volatility of the sectoral output gaps and the relative price gap, calling for policy intervention. In the following section, we visually show that these shocks cause large fluctuations in the sectoral output gaps and the relative price gap under the estimated monetary policy, and thus the optimal monetary policy alters the allocation in response to these shocks in a quantitatively important manner.

changed.

[7] Justiniano, Primiceri and Tambalotti (2013) show that without taking into account the measurement error in wage data, the role of the wage markup shock on wage inflation becomes large. Given the large welfare costs due to wage inflation, they show the optimal equilibrium is to have erroneously large fluctuations in the output gap to reduce wage inflation as much as possible.

5 The Relative Price Gap and Monetary Policy Trade-Off

In this section, we use our estimated model to simulate the historical relative price gap and associated inefficient fluctuations under the estimated monetary policy. We then explore how much of these fluctuations could have been avoided if monetary policy was optimal. To this end, we introduce three notions of equilibrium. Actual equilibrium represents an equilibrium under the estimated monetary policy. Efficient equilibrium represents an equilibrium under flexible prices and wages with zero markups. Optimal equilibrium represents an equilibrium under the optimal monetary policy. We focus on the optimal monetary policy under commitment, the path of the nominal interest rates that maximize the conditional welfare of the economy subject to the private agent's optimality conditions. We then combine the optimal and efficient equilibrium conditions with actual equilibrium conditions. The resulting system is linearized around the steady state and solved. Using the Kalman smoother, we compare the historical path of the sectoral output gaps, the sectoral price and wage inflation rates, and the relative price gap under the estimated and optimal policy.

As depicted in Figure 1, in the actual equilibrium, the relative price gaps are volatile, reflecting a large distortion in the relative price of investment. Volatile relative price gaps are associated with a volatile consumption-sector output gap (consumption gap) and an even more volatile investment-sector output gap (investment gap).[8] Moreover, consumption and investment gaps appear to comove negatively. Once consumption and investment gaps are HP-filtered with parameter 1600, the correlation between the two is -0.23. A positive relative price gap amounts to having the actual price of investment goods higher than its efficient level. As a result, households reduce the demand for investment goods and seek consumption goods. This process leads to the actual production of investment goods below its efficient level and the actual production of consumption goods above its efficient level. Moreover, sectoral price and wage inflation are inefficient due to the costs of price and wage dispersion.

In general, the distorted relative price of investment introduces a complex monetary policy trade-off among stabilizing the sectoral output gaps, sectoral price inflation, and sectoral wage inflation. First, the negative comovement of the sectoral output gaps makes it difficult for the monetary authority to stabilize both sectoral output gaps at the same time. For

[8] In Appendix C, we define gaps under the inefficient steady state assumption and find that the evolution of the gaps looks very similar to that in Figure 1.

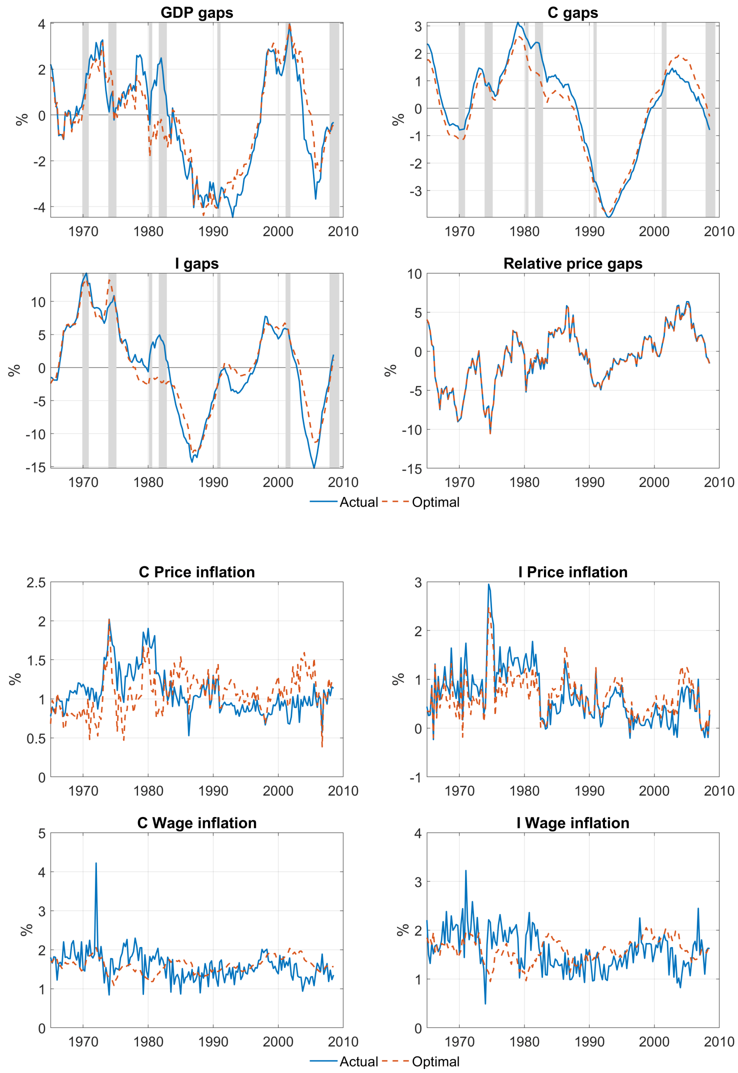

Figure 1. Actual and optimal gaps, sectoral price and wage inflation rates

example, raising the interest rate to choke off an inefficient boom in the consumption sector leads to a further reduction in the investment, implying a more negative investment gap. Likewise, an attempt to moderate a contractionary investment by lowering the real interest rate leads to more expansion in consumption, widening the consumption gap. Second, within a sector, stabilizing the output gap requires volatile price and wage inflation due to the presence of sector-specific markup shocks.

Given a possible complex trade-off among stabilizing the multiple objects, one might wonder whether such a trade-off faced by the monetary authority is quantitatively important. Table 3 compares the volatility of selected variables over the sample period under the estimated

Table 3. Performance of optimal monetary policy

Policy	The standard deviation					
	C gap	I gap	$\pi_{c,t}$	$\pi_{i,t}$	$\pi^w_{c,t}$	$\pi^w_{i,t}$
Estimated monetary policy rule	1.75	6.98	0.27	0.56	0.30	0.36
Optimal monetary policy	1.66	6.43	0.26	0.44	0.20	0.24
Percentage change (%)	5.14	7.88	3.70	21.43	33.33	33.33

Note: $\pi_{c,t}$ is the consumption price inflation rate. $\pi_{i,t}$ is the investment price inflation rate. $\pi^w_{c,t}$ is the consumption wage inflation rate. $\pi^w_{i,t}$ is the investment wage inflation rate. The measurement errors are excluded when computing the standard deviation of sectoral wage inflation. Row 3 represents the difference in the volatility of selected variables between the estimated monetary policy rule and optimal monetary policy in percentage change.

and optimal policy.[9] The variations in the sectoral output gaps, the sectoral price inflation rates, and the sectoral wage inflation rates are reduced under the optimal monetary policy, increasing the welfare of our model economy. However, the reduction of nominal and real fluctuations is unequal: the sectoral wage inflation rates are stabilized to a large extent, but the sectoral output gaps remain volatile. Figure 1 confirms the result by showing that the actual (solid line) and optimal (dashed line) sectoral output gaps roughly coincide, while the sectoral wage inflation rates appear to be quite stable in the optimal equilibrium. The limited joint nominal and real stabilization under optimal monetary policy indicate that the monetary policy trade-off is quantitatively significant.

One notable feature in Figure 1 is that the relative price gaps are essentially the same under the estimated and optimal policy. The equivalence arises due to near-complete factor mobility and almost identical nominal rigidity across sectors in our estimated model. The intuition is as follows. For the sake of exposition, assume a perfectly competitive labor market and flexible wages. When factors are completely mobile, factor prices are the same in both sectors. In this environment, the relative price gap is determined by the difference in the price markup between sectors, as seen in equation (4). The difference in the price markup between sectors can be decomposed into its exogenous and endogenous factors. The exogenous factors correspond to the difference between two sector-specific price markup shocks. When prices are equally sticky across sectors, the endogenous factors arise from the difference in nominal marginal costs between sectors. The difference in nominal marginal costs, in turn, arises from IST shocks as factor prices are the same across sectors. Therefore, the relative price gap is only a function of sector-specific price markup and IST shocks, which are unrelated to policy. Accordingly, the relative price gaps are identical under the estimated and

[9]For the optimal equilibrium, shocks to the estimated interest rate rule and target inflation are excluded as these shocks are replaced by optimal monetary policy.

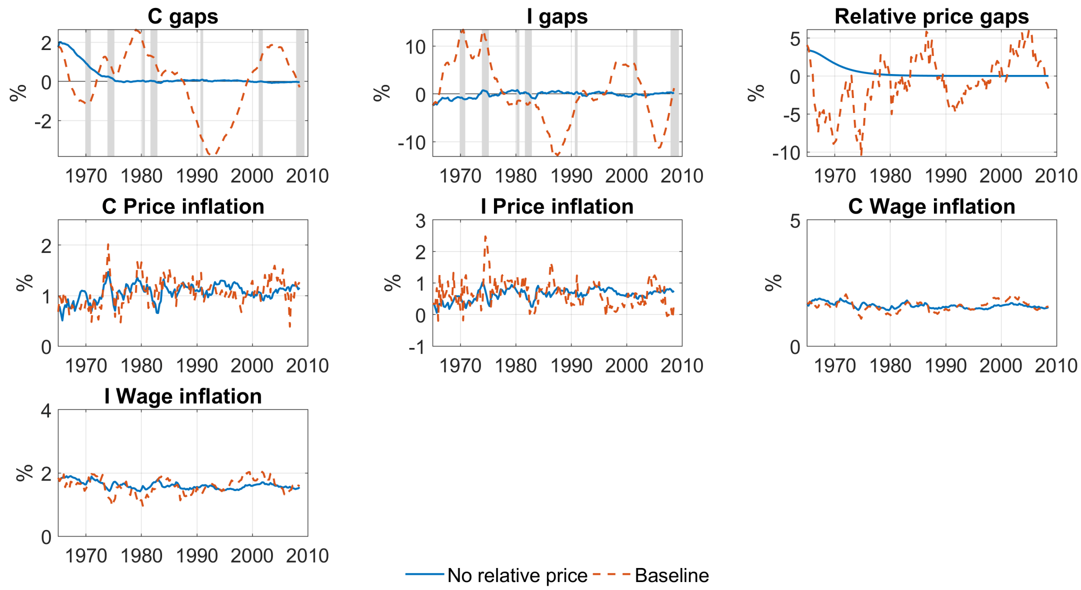

Figure 2. Optimal gaps and inflation under constant relative price of investment

optimal policy. In Appendix B, we analytically prove that the dynamics of the relative price gaps are determined by terms that are independent of policies when factors are completely mobile across sectors, and price stickiness is the same. Therefore, in our model, the optimal monetary policy alters the output gaps and the inflation dynamics in a way that maximizes the welfare taking the relative price gaps as given.

To highlight the importance of the fluctuations in the relative price gap in the monetary policy trade-off, we introduce a benchmark in which the relative price gap is zero. In our model, the relative price gap varies due to factor immobility, asymmetric nominal rigidity across sectors, the IST shock, and sector-specific price and wage markup shocks. Therefore, we assume complete factor mobility (i.e., $\omega^N = \omega^K = 0$) and equal sticky prices and wages (i.e., $\xi_{pc} = \xi_{pi} = 0.81$, $\iota_{pc} = \iota_{pi} = 0.14$, $\xi_{wc} = \xi_{wi} = 0.84$, $\iota_{wc} = \iota_{wi} = 0.26$). Furthermore, we shut down the IST shock, and sector-specific price and wage markup shocks to obtain a benchmark in which the relative price of investment is constant, implying a zero relative price gap. In this zero-relative price gap benchmark, the correlation between the HP-filtered consumption and investment gaps under the estimated monetary policy rule is 0.77, confirming that cyclical fluctuations in the relative price gap are the main drivers of the negative comovement between the consumption and investment gap. Figure 2 compares the historical optimal equilibrium path in the zero-relative price gap benchmark (solid line) and that in the baseline (dashed line). It shows that the optimal sectoral output gaps, sectoral price inflation rates, and sectoral wage inflation rates are very stable at all times in an economy with

Table 4. Variance decomposition under optimal monetary policy

Shock/series	GDP gap	C gap	I gap	RP gap	$\pi_{c,t}$	$\pi_{i,t}$	$\pi^w_{c,t}$	$\pi^w_{i,t}$
C price markup	50.13	51.19	1.91	12.19	68.00	0.25	4.07	3.50
I price markup	8.64	24.44	74.40	32.32	1.61	79.86	2.00	15.25
Aggregate TFP growth	1.43	0.13	1.54	0.05	15.39	2.28	24.92	26.24
IST growth	35.52	21.27	18.05	50.72	7.57	16.46	45.42	32.33
Aggregate TFP	0.86	0.13	0.56	0.00	4.57	0.51	3.86	6.27
MEI	1.80	2.03	1.65	4.14	2.18	0.36	9.49	4.81
Preference	0.25	0.08	0.47	0.08	0.01	0.01	1.25	3.42
C wage markup	0.55	0.17	0.26	0.01	0.03	0.01	3.21	1.36
I wage markup	0.73	0.16	0.67	0.03	0.04	0.02	1.76	6.6
Gov. consumption	0.05	0.02	0.00	0.02	0.05	0.00	0.00	0.13
Gov. investment	0.03	0.02	0.02	0.05	0.01	0.01	0.21	0.00
Labor supply	0.02	0.36	0.46	0.39	0.53	0.24	3.82	0.10

Note: Decomposition computed at the posterior mean using the Band-Pass filter with the pass band of 6-32 quarters. RP gap denotes the relative price gap. $\pi_{c,t}$ is the consumption price inflation rate. $\pi_{i,t}$ is the investment price inflation rate. $\pi^w_{c,t}$ is the consumption wage inflation rate. $\pi^w_{i,t}$ is the investment wage inflation rate.

zero relative price gaps. The stark difference in the evolution of real and nominal variables between the baseline and the counterfactual benchmark clearly shows that the distorted relative price is the key to the monetary policy trade-off.

The joint stabilization of real and nominal variables in the absence of the relative price gap confirms the findings of Justiniano, Primiceri and Tambalotti (2013). They show that in an estimated one-sector model, price inflation, wage inflation, and the output gap are roughly stabilized at the same time when monetary policy is optimal. In their model, the relative price of investment is not distorted, and thus the relative price gap is zero.

6 Sources of Limited Joint Real and Nominal Stabilization

As shown in Table 3, while the optimal monetary policy has difficulty in stabilizing the sectoral output gaps, it can reduce the volatility of the inflation rates, especially the sectoral wage inflation rates. In this section, we explore why joint stabilization of the output gaps and the inflation rates does not emerge in the optimal equilibrium.

To identify the shocks that contribute the most to the lack of joint stabilization of nominal and real variables, we present a variance decomposition at business-cycle frequencies under

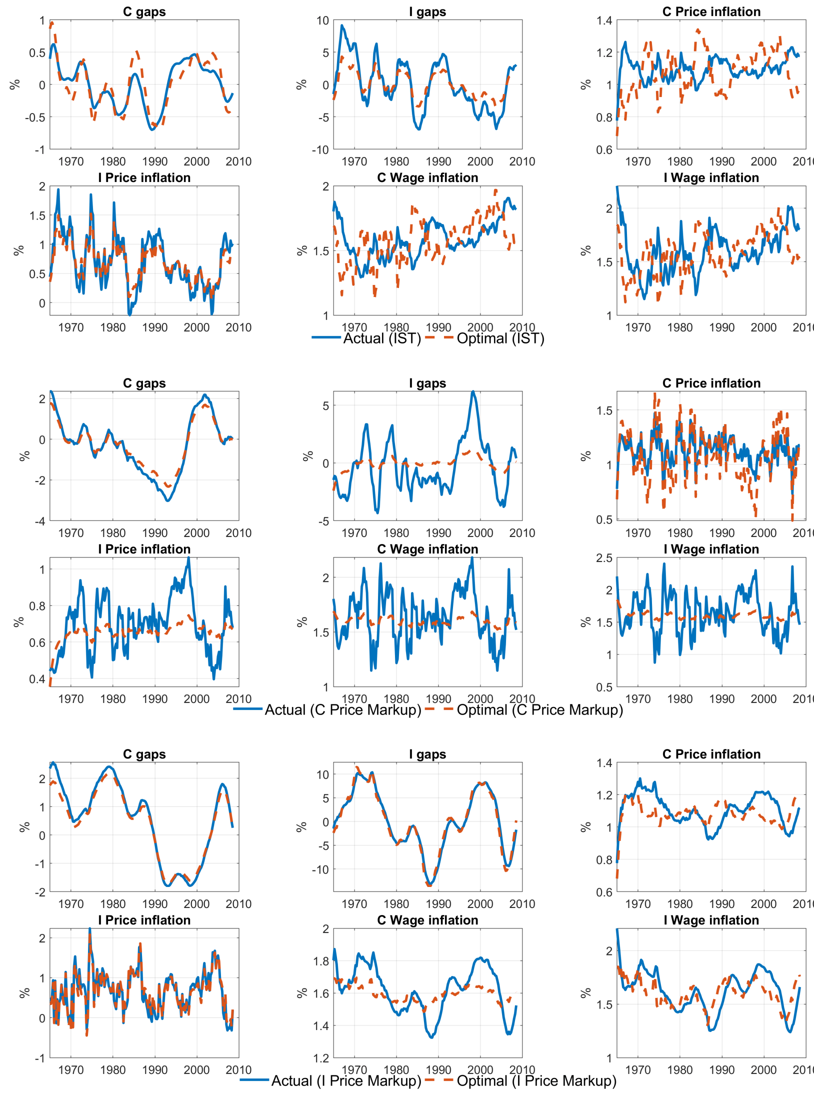

Figure 3. Actual and optimal gaps, sectoral price and wage inflation rates conditional on IST and sectoral price markup shocks

the optimal monetary policy, shown in Table 4. Shocks to consumption and investment-sector price markups and IST explain around 95 percent of the fluctuation in the optimal relative price gap. At the same time, these shocks explain roughly 95 percent of the fluctuation in the optimal consumption and investment gap. Therefore, the IST and sectoral price markup shocks are the major disturbances that prevent the joint stabilization of the output gaps and the inflation rates. Figure 3 compares the historical paths of the sectoral output gaps, price inflation, and wage inflation under the estimated and optimal monetary policy for each shock.

In response to the IST shock, the volatility of the consumption gap and consumption price inflation is increased under the optimal policy compared to the estimated policy, while the

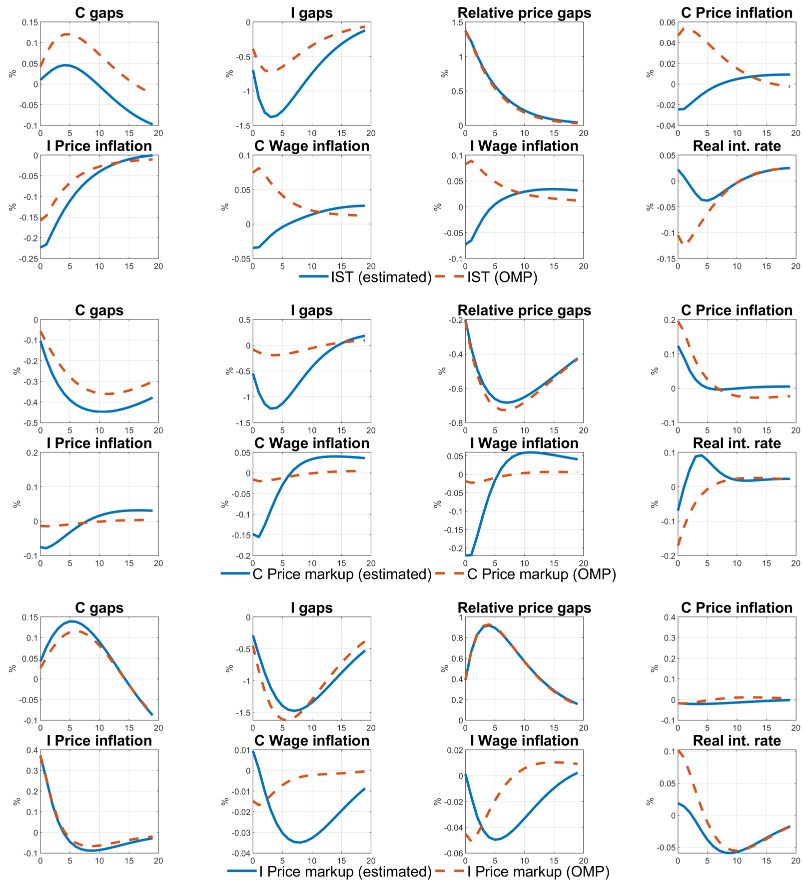

Figure 4. IRFs conditional on IST and sectoral price markup shocks

volatility of the investment gap and investment price inflation is reduced. These dynamics illustrate the trade-off between sectors. A more volatile consumption gap and consumption price inflation must be present to reduce the costly fluctuations in the investment gap and investment price inflation.

While the IST shock creates a trade-off among stabilizing variables between sectors, sector-specific price markup shocks create a more complex trade-off that includes a trade-off within the sector. In particular, in response to the consumption price markup shock, consumption price inflation in the optimal equilibrium is more unstable than in the actual equilibrium. This destabilization of consumption price inflation is the price that the monetary authority must pay to attain the reduced volatility in the consumption gap and the stabilization of consumption wage inflation. In addition, destabilized consumption price inflation is associated with stabilized investment price and wage inflation, indicating a between-sector trade-off. In the case of the investment price markup shock, reduced volatility in the consumption gap, consumption price inflation, and consumption wage inflation in the optimal equilibrium is accompanied by increased volatility in the investment gap, evidence of a between-sector trade-off. At the same time, a more volatile investment gap is a cost associated with achieving reduced variations in investment wage inflation, implying a within-sector trade-off.

Having discussed the properties of the optimal equilibrium, We now provide the intuition behind the trade-off induced by IST and price markup shocks. We do so by comparing impulse responses under the estimated monetary policy (solid lines) and the optimal monetary policy (dashed lines), displayed in Figure 4.

In response to a positive IST shock, the efficient relative price drops because a positive IST shock directly lowers nominal marginal costs in the investment sector. Because the investment sector prices are sticky, the relative price under the estimated policy falls less than its efficient counterpart, implying a positive relative price gap. A positive relative price gap means that investment goods in the actual equilibrium are more expensive than those in the efficient equilibrium. As a result, the actual demand for investment goods is below the efficient level, implying a negative investment gap. In addition, a negative relative price gap implies that consumption goods in the actual equilibrium are cheaper than those in the efficient equilibrium, resulting in a positive consumption gap.

The opposite movement between the consumption and investment gap is at the center of the between-sector monetary policy trade-off. On the one hand, investment goods need to be produced more, requiring the optimal monetary policy to keep the real interest rate below the level set by the estimated rule. To close the consumption gap, on the other hand, consumption goods need to be produced less, requiring the optimal real interest rates to be above the level set by the estimated rule. Because investment is much more sensitive to changes in the relative prices due to the high stock-flow ratio of capital, the optimal policy puts more weight on stabilizing the investment gap, inducing the optimal real interest rate to be below the estimated real rate. As a result, a higher consumption gap and a smaller fall in the investment gap are realized under the optimal policy compared to the estimated policy. A higher consumption level under the optimal policy induces households to supply hours worked below the level attained under the estimated policy putting an upward pressure on sectoral wage inflation. The trade-off that arises between sectors is emphasized by Erceg and Levin (2006) and Basu and De Leo (2019). We verify that this kind of trade-off is relevant in our estimated model.

As in the case with IST shocks, investment price markup shocks lead to the opposite movement between the consumption and investment gap under the estimated policy. An exogenous increase in the investment price markup causes the investment price under the estimated policy to be higher than its efficient level, leading to a positive relative price gap. This implies that investment goods under the estimated policy are more expensive than in the efficient equilibrium, generating a negative investment gap. At the same time, a positive relative price gap means that the consumption price under the estimated rule is lower than its efficient level, implying a positive consumption gap. However, a crucial difference between these two shocks is observed in the optimal equilibrium. A more volatile consumption gap and a less volatile investment gap under the optimal policy compared to the estimated policy is no longer observed when it comes to a positive investment price markup shock. A positive investment price markup shock causes the optimal policy to generate a less volatile consumption gap than the estimated policy and a more volatile investment gap.

Why are the responses to investment price markup shocks in the optimal equilibrium drastically different from those to IST shocks? A negative comovement between the sectoral output gaps introduces a between-sector monetary trade-off as in the case of a positive IST shock. If such a trade-off is the only challenge that the monetary authority faces, then the optimal policy is to stabilize the investment gap and allow a more volatile consumption gap by sufficiently lowering the real interest rate. However, such an allocation is not attained as the investment price markup shock introduces a within-sector trade-off. Specifically, the optimal policy chooses to stabilize investment wage inflation at the cost of a more volatile investment gap. This choice is achieved by having the optimal real interest rate rise more than the rate predicted under the estimated policy. The resulting optimal real rate dampens consumption, causing a less volatile consumption gap under the optimal policy than under the estimated policy. Due to near-perfect factor mobility, stable investment wage inflation is associated with stable consumption wage inflation.

In response to a positive consumption price markup, the consumption price under the estimated policy becomes higher than its efficient level leading to a negative relative price gap. This implies that consumption goods under the estimated policy are more expensive than in the efficient equilibrium, generating a negative consumption gap. Moreover, the hump-shaped fall in the relative price gap arises because of sticky investment prices. As some firms in the investment sector are not able to lower their prices, investment goods are more expensive today than in the future. The expected capital loss under the estimated rule suppresses the investment demand, implying a negative investment gap. If the monetary authority's only objective is to close the consumption and investment gap, the optimal real interest rate

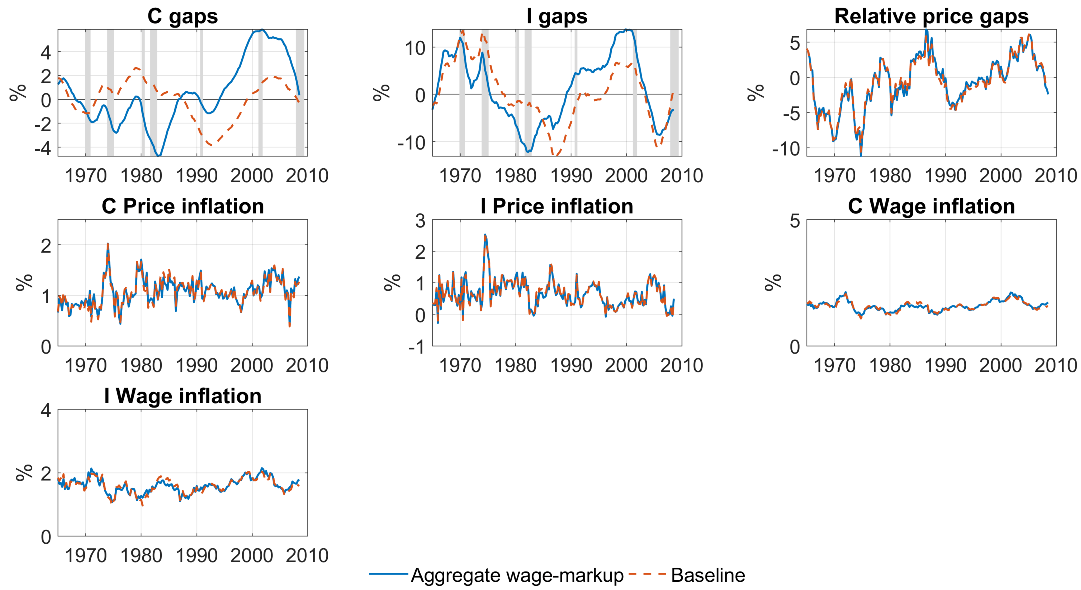

Figure 5. Effect of alternative assumptions on the origin of common labor supply on the optimal gaps and sectoral inflation rates

should be sufficiently below the real rate predicted by the estimated policy. However, the within-sector trade-off prevents complete stabilization of the real gaps. In particular, if the monetary authority prioritizes the stabilization of the consumption gap, its choice entails a very destabilized consumption price and wage inflation, which is costly. Therefore, the optimal policy strikes a balance between stabilizing the consumption gap and consumption price and wage inflation.

In sum, due to the trade-offs mentioned above, IST and both price markup shocks limit the stabilization of the sectoral output gaps in the optimal equilibrium. Both price markup shocks lead to stable wage inflation in the optimal equilibrium. In aggregate, the lack of joint stabilization of the output gaps and the inflation rates under the optimal policy illustrated in Figure 1 is mostly a consequence of the combined effect of historical shocks to IST and the sectoral price markup, with the effect of each shock depending on its estimated standard deviation.

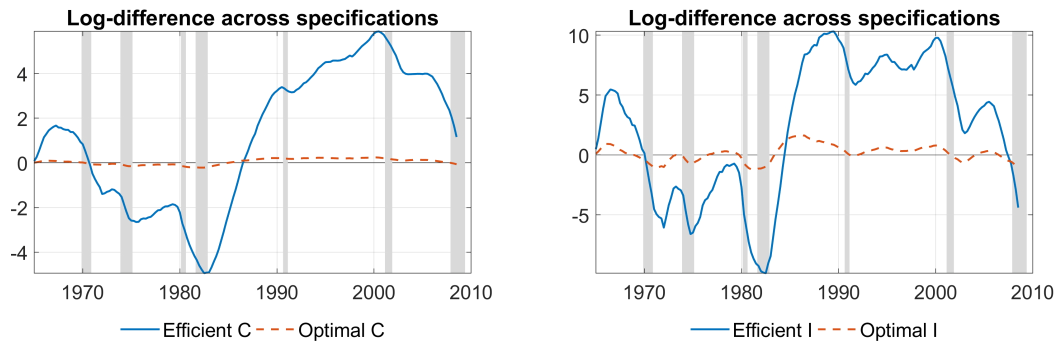

Figure 6. Effect of alternative assumptions on the origin of common labor supply on efficient and optimal consumption and investment.

7 Further Discussion

7.1 Choice of Shocks

We use the observable variables chosen by Moura (2018) for estimation. While the number of observable variables equals the number of shocks in Moura (2018), there are more shocks than observables in our model. The shocks that are present in our paper but are absent in Moura (2018) are the inflation target, stationary aggregate TFP, and aggregate labor supply shocks. In this subsection, we discuss the extent to which these shocks matter for our positive and normative results.

As justified in Justiniano, Primiceri and Tambalotti (2013), the persistent inflation target shock captures the low-frequency movement of consumption price inflation, which might arise from changes in preferences of monetary authority over inflation outcomes. Following Schmitt-Grohé and Uribe (2012), we include the stationary aggregate TFP shock. These two shocks may affect the inference of other shocks, especially the sectoral price markup shocks, that determine the extent to which the relative price of investment is distorted. Therefore, the inflation target and stationary aggregate TFP shocks might affect the volatility of the relative price gap and thus the extent to which the sectoral output gaps and the inflation rates are stabilized under the optimal monetary policy. For example, because the inflation target shock and the consumption price markup shock both affect the variations in consumption price inflation in our model, excluding the former might increase the estimated size of the latter. Similarly, excluding the stationary aggregate TFP shock eliminates the possibility of this shock explaining sectoral outputs and thus might increase the estimated size of sectoral price markup shocks. Our estimation result reveals that the contribution of the inflation target and stationary aggregate TFP shocks in explaining the sectoral output gaps and the relative price gap is virtually zero, as confirmed in Table 2. Therefore, these shocks do not affect the actual and optimal paths of the sectoral output gaps and the inflation rates meaningfully.[10]

Although the behavior of the sectoral output gaps is barely affected by the inflation target and stationary aggregate TFP shocks, it is significantly affected by the aggregate labor supply shock. The main purpose of introducing the aggregate labor supply shock, as in Justiniano, Primiceri and Tambalotti (2013), is to produce the dynamics of the GDP gap with reasonable cyclical properties. With this shock, the dynamics of the GDP gap predicted from our model are similar to those from their model.[11]

As mentioned in section 2, in our model, the aggregate labor supply shock is the source of the movements of the common component of sectoral labor supply. An observationally equivalent shock to the aggregate labor supply shock is the aggregate wage markup shock. Justiniano, Primiceri and Tambalotti (2013) demonstrate that while the positive implications of these two shocks are identical, their normative implications are not. We investigate whether this is the case in our two-sector model. Figure 5 compares the optimal paths of the sectoral output gaps and the inflation rates that arise from two alternative interpretations of the origin of fluctuations in common labor supply. The solid line in the figure represents the historical dynamics when the source of common labor supply is the aggregate wage markup shock.[12] The dashed line represents the dynamics when the source is the aggregate labor supply shock (baseline). While the shape of the sectoral output gaps is different under two alternative assumptions, the relative price gaps and the inflation rates are the same. The unchanged dynamics of the relative price gaps are not surprising because both shocks shift the sectoral nominal marginal cost almost uniformly because of almost equal wage stickiness across sectors. Given the almost identical price stickiness in our estimated model, the relative price is almost unchanged. Therefore, the response of the relative price gaps is close to zero conditional on these two shocks, and thus the unconditional dynamics of the relative price gaps are barely affected by these two shocks.

[10] We have verified this statement by simulating a model in which the inflation target and stationary aggregate TFP shocks are eliminated. It turns out that the actual and optimal paths of the sectoral output gaps and the inflation rates are very similar to those shown in Figure 1.

[11] We found that, in the absence of the aggregate labor supply shocks, the dynamics of the GDP gap are quite unrealistic, with the GDP gap being positive during the recent financial crisis.

[12] We re-estimated the model and found that the estimates of the structural parameters are virtually unchanged.

Table 5. Optimal weights on the sectoral inflation rates

Model specification	Welfare cost	Optimal weights	
		$\pi_{c,t}$	$\pi_{i,t}$
Flexible wages & IST shocks	0.10	0.37	0.63
Flexible wages & all shocks	0.28	0.53	0.47

Note: The welfare loss is expressed as a percent of steady-state consumption.

Why do the optimal sectoral output gaps differ under different assumptions on the source of common labor supply, whereas the sectoral inflation rates are the same? This is because the aggregate wage markup shock does not change the efficient allocation, whereas the aggregate labor supply shock does. Figure 6 illustrates this quantitatively by comparing the evolution of efficient consumption and investment arising from two alternative assumptions. The solid line represents the difference between efficient consumption and investment across the two scenarios, the aggregate labor supply shock (baseline) and the aggregate wage markup shock. The difference is very large and persistent. However, this is not the case for the optimal consumption and investment, as observed from the dashed line in Figure 6, which represents the difference between the optimal consumption and investment across the two scenarios. Moreover, as seen in Figure 5, the optimal inflation rates under the two scenarios are identical. The fact that the source of common labor supply shifts matters for the efficient fluctuations and not for the optimal fluctuations is consistent with the findings of Justiniano, Primiceri and Tambalotti (2013). As a result, the difference in the sectoral output gaps across the two alternative scenarios is mostly attributable to the difference in efficient allocation. Although the dynamics of the sectoral output gaps are altered when the aggregate labor supply shock is replaced with the aggregate wage markup shock, we emphasize that the consequences of the optimal monetary policy are unchanged. The output gaps are volatile, and inflation, especially wage inflation, is stable under the optimal policy.

7.2 Should Central Banks Target Investment Prices?

In this subsection, we discuss the implication of the distorted relative price for policymaking. This issue is studied in Basu and De Leo (2019) in the context of a calibrated two-sector model. More specifically, they ask whether the optimizing monetary authority should target both consumption and investment price inflation. They found that the answer is yes, and the optimal weight assigned to investment price inflation is significantly higher than the share of investment in GDP. Their result stems from the fact that investment displays an almost infinite intertemporal elasticity of substitution. Accordingly, changes in the relative price of investment lead to much larger fluctuations in the investment gap than the consumption gap. To moderate the costly investment gap volatility, the monetary authority should put a high weight on targeting investment price inflation.

In this subsection, we test their claim using our model which features more frictions and shocks than their model. To test their claim, we consider a composite inflation targeting policy that aims to stabilize the weighted average of the sectoral inflation rates. Formally, the composite gross inflation rate is

$$\pi_t = \pi_{c,t}^{\vartheta_{\pi_c}} \pi_{i,t}^{1-\vartheta_{\pi_c}},$$

where $\vartheta_{\pi_c} \in [0,1]$ is the weight of the consumption price inflation rate. We choose an optimal weight that yields the lowest welfare cost. If the optimal weight on investment price inflation $1-\vartheta_{\pi_c}$ is greater than 0.36, the steady state share of investment in GDP in our model, then the claim made by Basu and De Leo (2019) holds.

We derive the optimal weights in two benchmarks. The first benchmark assumes flexible wages and assumes that the IST shock is the only shock. Because the model of Basu and De Leo (2019) assumes flexible wages and only features sector-specific productivity shock, it would be fair to compare the results from this benchmark to theirs. As illustrated in the first row of Table 5, the optimal weight on investment price inflation is 0.63, which is significantly higher than the share of investment in GDP, verifying the argument made by Basu and De Leo (2019).[13]

The second benchmark assumes that all shocks are operative, and nominal wages are flexible. As seen in the second row of the table, this benchmark features more volatile inefficient fluctuations as many shocks are present, and thus the welfare cost of business cycles in this economy is higher than in the benchmark with only IST shocks. The optimal weight of investment price inflation in the second benchmark turns out to be 0.47, lower than the value in the first benchmark. As mentioned in the previous section, investment price markup shocks generate a trade-off within the investment sector. Assigning a large weight on investment price inflation entails large fluctuations in the investment gap, causing large social losses. Therefore, it is optimal to put less weight on investment price inflation in the economy with all shocks than the economy with only IST shocks.

[13]Notice that we include durable expenditures in investment, whereas Basu and De Leo (2019) do not. Therefore, the share of investment in GDP and the optimal weight on investment price inflation are higher than their computed values.

To summarize, the argument of Basu and De Leo (2019) generally holds in our quantitative model, but the optimal weight of investment price inflation heavily depends on the nature of shocks.

8 Conclusion

Most existing studies assume that the fluctuations in the relative price of investment are efficient. In this paper, we decomposed the fluctuations in the relative price of investment into its efficient and inefficient movements using an estimated two-sector New Keynesian model. We found that a large part of the variations in the observed relative price is inefficient, summarized by a volatile relative price gap. Fluctuations in the relative price gap are associated with a negative comovement between the sectoral output gaps. A significant degree of sectoral price and wage stickiness in our estimated model implies that sectoral price and wage inflation constitute additional sources of inefficiencies. Therefore, the monetary authority confronts a complex trade-off among the stabilization of the sectoral output gaps, sectoral price inflation, and sectoral wage inflation.

To evaluate the quantitative importance of the trade-off, we computed the evolution of the counterfactual economy that would be observed if a central bank follows the optimal monetary policy. We found that while the sectoral inflation rates, especially the wage inflation rates, are stabilized under the optimal monetary policy, a large variation in the sectoral output gaps remains.

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A Data Description

This appendix describes the construction of observable variables used in estimation. We define nominal private consumption as nominal consumption expenditure on nondurable goods and services (BEA, NIPA Table 1.1.5, lines 5 and 6), and nominal private investment as the sum of nominal consumption expenditures on durable goods and nominal fixed investment (NIPA Table 1.1.5, lines 4 and 8). Nominal government consumption expenditures and nominal government investment are retrieved from the BEA (NIPA Table 3.9.5, lines 2 and 3). In order to construct the consumption price index ($P_{c,t}$), we combine the price indices of nondurable goods and services (BEA, NIPA Table 1.1.4, lines 5 and 6) by chain aggregation. Similarly, for the investment price index ($P_{i,t}$), we combine the price indices of durable goods and fixed investment (BEA, NIPA Table 1.1.4, lines 4 and 8) by chain aggregation. The real private consumption and investment and real government consumption and investment are constructed by deflating each nominal series by the corresponding chain-aggregated price index. All quantity series are converted to per-capita terms using the population series provided by the BEA (NIPA Table 2.1, line 40).

Inflation and the relative price of investment Inflation in the consumption sector is defined as the growth rate in the chain-aggregated consumption price index. The relative price of investment goods is defined as $P_{i,t}/P_{c,t}$.

Hours worked We obtain series on employees and average hours worked for the total private industry (CES0500000001, CES0500000007), construction (CES2000000001, CES2000000007), durable manufacturing (CES3100000001, CES3100000007), and professional and business services (CES6000000001, CES6000000007) from the FRED. For each industry, we compute total hours as the product of employees and average hours. We define investment hours $N_{i,t}$ as the sum of hours worked in construction, durable manufacturing, and professional and business services. We then obtain consumption hours $N_{c,t}$ as the difference between total private hours and investment hours. Lastly, both hours series are detrended using a quadratic filter, as consumption and investment hours rise at a different rate over the sample.

Real wages We obtain series on nominal average hourly earnings for each of the above industries from the FRED. For each industry, we then compute the aggregate wage bill as the product of nominal average hourly earnings and total hours. We allocate the aggregate wage bill for the total private industry to the consumption and investment sector, using the same classification used for hours worked. For each sector, we compute the nominal wage rates by dividing the aggregate wage bills by hours worked. Finally, real wages for each sector are obtained by dividing the sectoral nominal wage rates by the consumption price index $P_{c,t}$.

Interest rate The nominal interest rate is the quarterly average of the effective Federal Funds rate, which is retrieved from the FRED.

B Dynamics of the Relative Price Gap

Here, we analytically show that the response of the relative price gap is irrelevant to policy rules under the assumption of perfect factor mobility and equal price stickiness across sectors. To maximize tractability, we make further assumptions: a perfectly competitive labor market, no inflation indexation, and no stochastic and deterministic trends. We define $\widehat{x}_t = \log(x_t/x)$, where x_t is a variable in our model in time t and x is the steady value of x_t. The log-linearized Phillips curves are

$$\widehat{\pi}_{c,t} = \beta \mathbb{E}_t \widehat{\pi}_{c,t+1} + \Theta_{pc} \widehat{mc}_{c,t} + \widehat{\eta}^{p*}_{c,t} \tag{B.1}$$

$$\widehat{\pi}_{i,t} = \beta \mathbb{E}_t \widehat{\pi}_{i,t+1} + \Theta_{pi} (\widehat{mc}_{i,t} - \widehat{Q}_t) + \widehat{\eta}^{p*}_{i,t}, \tag{B.2}$$

where $\widehat{\eta}^{p*}_{j,t} \equiv \frac{(1-\xi_{pj})(1-\beta\xi_{pj})}{\xi_{pj}} \widehat{\eta}^{p}_{j,t}$ and $\Theta_{pj} = \frac{(1-\xi_{pj})(1-\beta\xi_{pj})}{\xi_{pj}}$ for $j = c, i$. $mc_{j,t}$ denotes the sector j real marginal cost in consumption units. Using the definition of the relative price of investment $Q_t = \frac{P_{i,t}}{P_{c,t}}$, the log-linearzed relative price of investment evolves according to

$$\widehat{Q}_t = \widehat{Q}_{t-1} + \widehat{\pi}_{i,t} - \widehat{\pi}_{c,t}. \tag{B.3}$$

Using the expression for the log-linearized relative price of investment, the dynamics of the log-linearized relative price gap $\widetilde{Q} = \log(Q_t/Q^e_t)$ can be written as

$$\widetilde{Q}_t = \widetilde{Q}_{t-1} + \widehat{\pi}_{i,t} - \widehat{\pi}_{c,t} - \Delta \widehat{Q}^e_t. \tag{B.4}$$

The relative nominal marginal cost is $\frac{MC_{i,t}}{MC_{c,t}} = \frac{1}{z^{1-\alpha}_{i,t}} \left(\frac{W_{i,t}}{W_{c,t}}\right)^{1-\alpha} \left(\frac{R^k_{i,t}}{R^k_{c,t}}\right)^{\alpha} = \frac{1}{z^{1-\alpha}_{i,t}} = Q^e_t$, where the use of expression (3) has been made in the last equality. The log-linearized relative real marginal cost in turn is

$$\widehat{mc}_{i,t} - \widehat{mc}_{c,t} = \widehat{Q}^e_t. \tag{B.5}$$

Subtracting the consumption Phillips curve from its investment counterpart yields

$$\widehat{\pi}_{i,t} - \widehat{\pi}_{c,t} = \beta \mathbb{E}_t(\widehat{\pi}_{i,t+1} - \widehat{\pi}_{c,t+1}) - \Theta_{pc}\widetilde{Q}_t + \widehat{\eta}^{p*}_{i,t} - \widehat{\eta}^{p*}_{c,t},$$

where the use of expression (B.5) has been made in equality. Using expression (B.4), the equation above can be rearranged as

$$\widetilde{Q}_t - \widetilde{Q}_{t-1} + \Delta\widehat{Q}^e_t = \beta \mathbb{E}_t(\widetilde{Q}_{t+1} - \widetilde{Q}_t + \Delta\widehat{Q}^e_{t+1}) - \Theta_{pc}\widetilde{Q}_t + \widehat{\eta}^{p*}_{i,t} - \widehat{\eta}^{p*}_{c,t}.$$

This can be further rearranged to the following second-order linear difference equation:

$$\mathbb{E}_t\widetilde{Q}_{t+1} - \frac{(1+\beta+\Theta_{pc})}{\beta}\widetilde{Q}_t + \frac{1}{\beta}\widetilde{Q}_{t-1} = -\mathbb{E}_t\Delta\widehat{Q}^e_{t+1} + \frac{1}{\beta}\Delta\widehat{Q}^e_t - \frac{1}{\beta}(\widehat{\eta}^{p*}_{i,t} - \widehat{\eta}^{p*}_{c,t}). \tag{B.6}$$

The next step is to rewrite the equation using the lag operator $\mathbf{L}$ as

$$\mathbb{E}_t\left(\mathbf{L}^{-2} - \frac{(1+\beta+\Theta_{pc})}{\beta}\mathbf{L}^{-1} + \frac{1}{\beta}\right)\widetilde{Q}_{t-1} = -\mathbb{E}_t\Delta\widehat{Q}^e_{t+1} + \frac{1}{\beta}\Delta\widehat{Q}^e_t - \frac{1}{\beta}(\widehat{\eta}^{p*}_{i,t} - \widehat{\eta}^{p*}_{c,t}), \tag{B.7}$$

where $\mathbf{L}Q_t = Q_{t-1}$ and $\mathbf{L}^2 Q_{t+1} = Q_{t-1}$. Define the characteristic equation as

$$\phi^2 - \frac{(1+\beta+\Theta_{pc})}{\beta}\phi + \frac{1}{\beta} = 0.$$

This equation has two real roots ϕ_1 and ϕ_2, satisfying $\phi_1 + \phi_2 = \frac{(1+\beta+\Theta_{pc})}{\beta}$ and $\phi_1\phi_2 = \frac{1}{\beta}$. Clearly, one root is larger than 1 in absolute value and the other is less than 1 in absolute value. We then factorize the left-hand side of (B.7) to derive

$$(\mathbf{L}^{-1} - \phi_1)(\mathbf{L}^{-1} - \phi_2)\widetilde{Q}_{t-1} = -\mathbb{E}_t\Delta\widehat{Q}^e_{t+1} + \frac{1}{\beta}\Delta\widehat{Q}^e_t - \frac{1}{\beta}(\widehat{\eta}^{p*}_{i,t} - \widehat{\eta}^{p*}_{c,t}). \tag{B.8}$$

We assume that $|\phi_1| < 1$ and $|\phi_2| > 1$. Then, it follows from (B.8) that

$$(\mathbf{L}^{-1} - \phi_1)\widetilde{Q}_{t-1} = \frac{-\mathbb{E}_t\Delta\widehat{Q}^e_{t+1} + \frac{1}{\beta}\Delta\widehat{Q}^e_t - \frac{1}{\beta}(\widehat{\eta}^{p*}_{i,t} - \widehat{\eta}^{p*}_{c,t})}{(\mathbf{L}^{-1} - \phi_2)} = \frac{\mathbb{E}_t\Delta\widehat{Q}^e_{t+1} - \frac{1}{\beta}\Delta\widehat{Q}^e_t + \frac{1}{\beta}(\widehat{\eta}^{p*}_{i,t} - \widehat{\eta}^{p*}_{c,t})}{\phi_2(1 - \phi_2^{-1}\mathbf{L}^{-1})}.$$

We then obtain

$$\begin{aligned}
\widetilde{Q}_t &= \phi_1 \widetilde{Q}_{t-1} + \frac{1}{\phi_2} \sum_{s=0}^{\infty} \phi_2^{-s} \mathbf{L}^{-s} (\mathbb{E}_t \Delta \widehat{Q}^e_{t+1} - \frac{1}{\beta} \Delta \widehat{Q}^e_t + \frac{1}{\beta} (\widehat{\eta}^{p*}_{i,t} - \widehat{\eta}^{p*}_{c,t})) \\
&= \phi_1 \widetilde{Q}_{t-1} + \frac{1}{\phi_2} \sum_{s=0}^{\infty} \phi_2^{-s} (\mathbb{E}_t \Delta \widehat{Q}^e_{t+1+s} - \frac{1}{\beta} \Delta \widehat{Q}^e_{t+s} + \frac{1}{\beta} (\widehat{\eta}^{p*}_{i,t+s} - \widehat{\eta}^{p*}_{c,t+s})) \\
&= \phi_1 \widetilde{Q}_{t-1} + \frac{1}{\phi_2} \sum_{s=0}^{\infty} \phi_2^{-s} (\mathbb{E}_t (\alpha - 1) \Delta \widehat{z}_{i,t+1+s} - \frac{(\alpha-1)}{\beta} \Delta \widehat{z}_{i,t+s} + \frac{1}{\beta} (\widehat{\eta}^{p*}_{i,t+s} - \widehat{\eta}^{p*}_{c,t+s})).
\end{aligned}$$

Given an initial value $\widetilde{Q}_0$, this expression gives the solution to (B.6). Clearly, the dynamics of the relative price gap only depend on IST and price markup shocks, which are the terms independent of policy.

C Additional Tables and Figures

Table A.1 reports the prior and posterior distributions for parameters governing the shock processes, which are omitted in the body of the paper.

In the body of the paper, we used constant labor and capital income tax rates to correct the distortions that arise from the monopolistic competition. Therefore, the steady state allocation in our economy was efficient. The gaps were defined as follows. Taking the relative price gap as an example, the relative price gap was $\log(Q_t/Q) - \log(Q^e_t/Q^e)$, where $Q = Q^e$. In this appendix, we relax the efficient steady state assumption by imposing zero tax rates. In this case, the steady state allocation in our economy is inefficient due to the presence of monopolistic competition. Then the relative price gap is defined as $\log(Q_t/Q) - \log(Q^e_t/Q^e)$, where $Q \neq Q^e$. Figure A.1 plots the actual and optimal sectoral output gaps, sectoral price inflation, and sectoral wage inflation under a distorted steady state assumption.

Table A.1. Prior and posterior distribution of shock processes

Parameter	Description	Prior dist.			Posterior dist.		
		Distribution	Mean	SD	Mean	5%	95%
$\rho_{\eta_{pc}}$	Auto. C price markup	Beta	0.6	0.2	0.94	0.90	0.98
$\rho_{\eta_{pi}}$	Auto. I price markup	Beta	0.6	0.2	0.76	0.67	0.86
ρ_{μ_z}	Auto. N tech. growth	Beta	0.4	0.2	0.59	0.45	0.73
ρ_{μ_i}	Auto. I tech. growth	Beta	0.4	0.2	0.06	0.01	0.10
ρ_A	Auto. N tech.	Beta	0.6	0.2	0.95	0.93	0.98
ρ_υ	Auto. MEI	Beta	0.6	0.2	0.76	0.70	0.83
ρ_ζ	Auto. preference	Beta	0.6	0.2	0.36	0.20	0.51
ρ_{gc}	Auto. gov. cons.	Beta	0.6	0.2	0.95	0.93	0.98
ρ_{gi}	Auto. gov. invest.	Beta	0.6	0.2	0.96	0.93	0.98
ρ_b	Auto. labor supply	Beta	0.6	0.2	0.96	0.94	0.98
$\rho_{\eta_{wc}}$	Auto. C wage markup	Beta	0.6	0.2	0.47	0.18	0.75
$\rho_{\eta_{wi}}$	Auto. I wage markup	Beta	0.6	0.2	0.65	0.32	0.99
$100\sigma_{\eta_{pc}}$	Std C price markup	Inv. Gamma	0.15	1	0.07	0.05	0.08
$100\sigma_{\eta_{pi}}$	Std I price markup	Inv. Gamma	0.15	1	0.16	0.13	0.19
$100\sigma_{\mu_z}$	Std N tech. growth	Inv. Gamma	1	1	0.49	0.37	0.60
$100\sigma_{\mu_i}$	Std I tech. growth	Inv. Gamma	1	1	2.13	1.93	2.31
$100\sigma_A$	Std N tech.	Inv. Gamma	1	1	0.52	0.45	0.59
$100\sigma_\upsilon$	Std MEI	Inv. Gamma	0.5	1	5.94	4.60	7.21
$100\sigma_\zeta$	Std preference	Inv. Gamma	1	1	2.62	2.04	3.18
$100\sigma_{\eta_{wc}}$	Std C wage markup	Inv. Gamma	0.05	0.03	0.03	0.02	0.05
$100\sigma_{\eta_{wi}}$	Std I wage markup	Inv. Gamma	0.05	0.03	0.05	0.02	0.08
$100\sigma_m$	Std mon. policy	Inv. Gamma	0.15	1	0.23	0.21	0.26
$100\sigma_{gc}$	Std gov. cons.	Inv. Gamma	0.5	1	1.11	1.00	1.23
$100\sigma_{gi}$	Std gov. invest.	Inv. Gamma	0.5	1	2.45	2.23	2.67
$100\sigma_b$	Std labor supply	Inv. Gamma	1	1	1.87	1.39	2.33
$100\sigma_{e_{wc}}$	Std C wage m. error	Inv. Gamma	0.5	1	0.24	0.21	0.27
$100\sigma_{e_{wi}}$	Std I wage m. error	Inv. Gamma	0.5	1	0.18	0.14	0.21
$100\sigma_\pi$	Std inflation target	Inv. Gamma	0.05	0.03	0.03	0.02	0.03

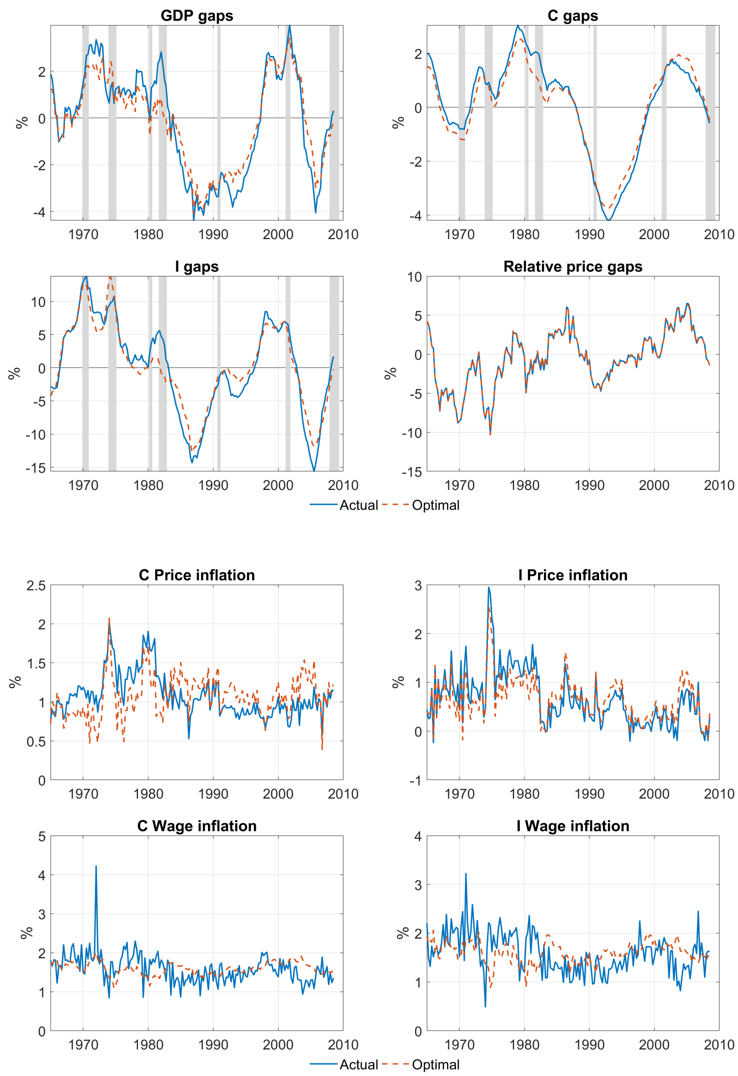

Figure A.1. Actual and optimal gaps, sectoral price and wage inflation rates under the distorted steady state